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Department: COMPUTER ENGINEERING | Course: MAT 104 | Serial No: 03

Assignment

$$1) y = \sin\left(\frac{6x}{x^2}\right)$$

$$y = \sin(6x^{-2})$$

$$y + Dy = \sin 6(x+Dx)^{-2}$$

$$y + Dy = \sin(6x^{-2} + 6Dx^{-2})$$

$$Dy = \sin(6x^{-2} + 6Dx^{-2}) - \sin 6x^{-2}$$

$$\frac{2\cos(A+B)}{2} \cdot \frac{\sin(A-B)}{2}$$

$$\frac{2\cos(6x^{-2} + 6Dx^{-2} + 6x^{-2})}{2} \cdot \frac{\sin(6x^{-2} + 6Dx^{-2} - 6x^{-2})}{2}$$

$$= \frac{2\cos(12x^{-2} + 6Dx^{-2})}{2} \cdot \frac{\sin(6Dx^{-2})}{2}$$

$$= \frac{2\cos(12x^{-2} + 6Dx^{-2})}{2} \cdot \frac{\sin(6Dx^{-2})}{2}$$

Divide both sides by Dx

$$\frac{Dy}{Dx} = \frac{2\cos(12x^{-2} + 6Dx^{-2})}{2} \cdot \frac{\sin(6Dx^{-2})}{2}$$

$$\lim_{Dx \rightarrow 0} = \frac{\cos(12x^{-2} + 6(0)^{-2})}{2} \cdot \frac{\sin 6Dx^{-2}}{2Dx^{-2}} \rightarrow 1$$

$$\frac{Dy}{Dx} = \frac{\cos 12x^{-2}}{2}$$

$$\therefore \frac{Dy}{Dx} = \cos 6x^{-2}$$

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2) $y = t^4 + 2t^2$, $x = 6t^3 - t^2$ at $t = 1$ and $t = 3$

Let A represent the area, then

$$A = \int_a^b y dx$$

$$y = t^4 + 2t^2$$

$$A = \int_a^b t^4 + 2t^2 dx$$

Given that $x = 6t^3 - t^2$

$$\frac{dx}{dt} = 18t^2 - 2t$$

$$dx = (18t^2 - 2t) dt$$

$$A = \int_1^3 (t^4 + 2t^2)(18t^2 - 2t) dt$$

$$A = \int_1^3 (18t^6 - 2t^5 + 36t^4 - 4t^3) dt$$

$$= \left[\frac{18t^7}{7} - \frac{2t^6}{6} + \frac{36t^5}{5} - \frac{4t^4}{4} \right]_1^3$$

$$= \left[\frac{18t^7}{7} - \frac{t^6}{3} + \frac{36t^5}{5} - t^4 \right]_1^3$$

$$= \left[\frac{18(3)^7}{7} - \frac{(3)^6}{3} + \frac{36(3)^5}{5} - (3)^4 \right] - \left[\frac{18}{7} - \frac{1}{3} + \frac{36}{5} - 1 \right]$$

$$= \frac{160704}{35} - \frac{544}{105}$$

$$= 4586.26 \text{ sq units}$$

$$3) x = 4t^3 - t^2, y = t^4 + 2t^2 \quad \text{find } \frac{dy}{dx}$$

$$x = 4t^3 - t^2$$

$$y = t^4 + 2t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t} \times 1$$

$$12t^2 - 2t$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$= \frac{4(t^3 + t)}{2(6t^2 - t)}$$

$$= \frac{2(t^3 + t)}{6t^2 - t}$$

$$\frac{dy}{dx} = \frac{2t^3 + 2t}{6t^2 - t}$$