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Chemical Engineering
MAT 104 Assignment

1 $y = \sin(6/x^2)$

Let $u = 6/x^2$

$\Rightarrow y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(\frac{2u+\Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u/2} \Rightarrow \frac{\cos\left(\frac{2u+\Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)}{\Delta u/2}$$

$$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u+\Delta u}{2}\right)\right] \cdot \lim_{\Delta u \to 0}\left[\frac{\sin(\Delta u/2)}{\Delta u/2}\right]$$

$\therefore \frac{dy}{du} = \cos u$

From eqn: $u = 6/x^2$

$$u + \Delta u = \frac{6}{(x + \Delta x)^2}$$

$$u + \Delta u = \frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x\Delta x + (\Delta x)^2} - u$$

$$\Delta u = \frac{6}{x^2 + 2x\Delta x + (\Delta x)^2} - \frac{6}{x^2}$$

$$\Rightarrow \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

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$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right] = \frac{-12x}{x^4}$$

$$\therefore \frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$$

$$\therefore \frac{dy}{dx} = \frac{-12\cos(6/x^2)}{x^3}$$

2 $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

$$A = \int_a^b y\,dx$$

$$= \int_1^3 (t^4 + 2t^2)\,dx$$

Given $x = 4t^3 - t^2$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$\Rightarrow A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$$

$$A = \left[\frac{12}{7}t^7 - \frac{2}{6}t^6 + \frac{24}{5}t^5 - \frac{4}{4}t^4\right]_1^3$$

$$= \left[\frac{12}{7}(3)^7 - \frac{2}{6}(3)^6 + \frac{24}{5}(3)^5 - (3)^4\right] - \left[\frac{12}{7}(1)^7 - \frac{2}{6}(1)^6 + \frac{24}{5}(1)^5 - (1)^4\right]$$

$$= \left[\frac{26244}{7} - 243 + \frac{5832}{5} - 81\right] - \left[\frac{12}{7} - \frac{2}{6} + \frac{24}{5} - 1\right]$$

$$= \frac{160704}{35} - \frac{649}{105}$$

$\Rightarrow A = 4585.36$ square units

3 $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dy}{dt} = 4t^3 + 4t \; ; \; \frac{dx}{dt} = 12t^2 - 2t \Rightarrow \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\Rightarrow \frac{dy}{dx} = (4t^3 + 4t) \cdot \left(\frac{1}{12t^2 - 2t}\right)$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t} = \frac{\cancel{4}^{2}(t^3 + t)}{\cancel{2}(6t^2 - t)} = \frac{2(t^3 + t)}{6t^2 - t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2t^3 + 2t}{6t^2 - t}$$