

Scanned Documents

FAGBAMILA GIDEON

MECHANICAL ENGINEERING

MAT 102

19/ENG06/024

1) find a Unit vector tangent to the Space Curve $x=t, y=t^2, z=t^3$ at the point where $t=1$

$$r'(t) = [1, 2t, 3t^2]$$

$$\|r'\| = \sqrt{(1)^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$$

$$\vec{T} = \frac{r'(t)}{\|r'\|} = \frac{[1, 2t, 3t^2]}{\sqrt{1 + 4t^2 + 9t^4}}$$

$$= \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} [1, 2t, 3t^2]$$

$$= \frac{1}{\sqrt{1 + 4(1)^2 + 9(1)^2}} [1, 2(1), 3(1)] = \frac{1}{\sqrt{1 + 4 + 9}} [1, 2, 3]$$

$$= \frac{1}{\sqrt{14}} [1, 2, 3] = \frac{1}{\sqrt{14}} \hat{i} + \frac{2}{\sqrt{14}} \hat{j} + \frac{3}{\sqrt{14}} \hat{k}$$

2) $A = 4t^3 \hat{j} + 5 \hat{k}, B = 2t^2 \hat{i} + 4t \hat{j}$

$$C = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4t^3 & 5 \\ 2t^2 & 4t & 0 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 4t^3 & 5 \\ 4t & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 0 & 5 \\ 2t^2 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & 4t^3 \\ 2t^2 & 4t \end{vmatrix}$$

$$C = -20t \hat{i} + 10t^2 \hat{j} - 8t^5 \hat{k}$$

$$\int C = \int -20t \hat{i} + 10t^2 \hat{j} - 8t^5 \hat{k} dt$$

$$= -10t^2 \hat{i} + \frac{10}{3} t^3 \hat{j} - \frac{8}{6} t^6 \hat{k}$$

$$\int_0^1 C = \left[-10(1)^2 \hat{i} + \frac{10}{3} (1)^3 \hat{j} - \frac{8}{6} (1)^6 \hat{k} \right] - \left[-10(0)^2 \hat{i} + \frac{10}{3} (0)^3 \hat{j} - \frac{8}{6} (0)^6 \hat{k} \right]$$

$$= -10 \hat{i} + \frac{10}{3} \hat{j} - \frac{4}{3} \hat{k}$$