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Mechanical engineering.
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MAT 104
NO 113

$$y = \sin\left(\frac{6}{x^2}\right), \quad y + \Delta y = \sin\left(\frac{6}{x + \Delta x}\right)^2$$

$$\Delta y = \sin\frac{6}{(x+\Delta x)^2} - y = \sin\frac{6}{(x+\Delta x)^2} - \sin\left(\frac{6}{x^2}\right)$$

$$\Delta y = 2\cos\left(\frac{\frac{6}{(x+\Delta x)^2} + \frac{6}{x^2}}{2}\right)\sin\left(\frac{\frac{6}{(x+\Delta x)^2} - \frac{6}{x^2}}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{\frac{6}{x^2 + 2x\Delta x + (\Delta x)^2} + \frac{6}{x^2}}{2}\right)\sin\left(\frac{\frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{\frac{6x^2 + 6x^2 + 12x\Delta x + 6(\Delta x)^2}{x^2(x^2 + 2x\Delta x + (\Delta x)^2)}}{2}\right)\sin\left(\frac{\frac{-12x\Delta x - 6(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}}{2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{6(2x^2 + 2x\Delta x + (\Delta x)^2)}{2(x^4 + 2x^3\Delta x + x^2(\Delta x)^2)}\right)\sin\left(6\left(\frac{-2x\Delta x - (\Delta x)^2}{2(x^4 + 2x^3\Delta x + x^2(\Delta x)^2)}\right)\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{6x^2 + 6x\Delta x + 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right)\frac{\sin\left(\frac{-6x\Delta x - 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{6x^2 + 6x\Delta x + 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right)\sin\left(\frac{-6x - 3\Delta x}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right)$$

$$\times \frac{-6x - 3\Delta x}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2} \div \frac{-6x - 3\Delta x}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}$$

Take $\lim\limits_{\Delta x \to 0}$

$$\frac{\Delta y}{\Delta x} = \frac{\left[2\cos\left(\frac{6x}{x^4}\right)\sin\left(\frac{-6x}{x^4}\right)\right]\left(\frac{-6x}{x^4}\right)}{-6x/x^4}$$

$$\frac{\Delta y}{\Delta x} = \left[2\cos\left(\frac{6x}{x^4}\right)\sin\left(\frac{-6x}{x^4}\right)\right]$$

2 Area (A) $= \int_a^b y\,dx$ where: $x = 4t^3 - t^2$, $y = t^4 + 2t^2$, at $t = 1$ and $t = 3$

$\frac{dx}{dt} = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$A = \frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}\Big|_1^3 = \frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4\Big|_1^3$

$A = \left[\frac{12(3)^7}{7} - \frac{1}{3}(3)^6 + \frac{24(3)^5}{5} - 3^4\right] - \left[\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right]$

$\therefore$ Area under the curve $= \frac{160536}{35} = 4586.74$.

3 $x = 4t^3 - t^2$ $\qquad y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$ $\qquad \frac{dy}{dt} = 4t^3 - 4t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4t^3 + 4t}{12t^2 - 2t} = \frac{2(2t^3 + 2t)}{2(6t^2 - t)} = \frac{2t(2t^2 + 2)}{2t(6t - 1)}$

$\frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}$

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