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1. A vector space over a field is a set that is closed under finite vector addition and scalar multiplication for V to be a vector space.
2. A=(1,1,1)
   B=(1,2,3)
   C=(1,5,8)
   Solution

$$\alpha\begin{bmatrix}1\\1\\1\end{bmatrix}+\beta\begin{pmatrix}1\\2\\3\end{pmatrix}+\delta\begin{pmatrix}1\\5\\8\end{pmatrix}=\begin{pmatrix}a\\b\\c\end{pmatrix}$$

=$\alpha+\beta+\delta=a$ …….1

$\alpha+2\beta+5\delta$=b………2

$\alpha+3\beta+8\delta$=c………..3

α=a-β-δ=c………..4

sub 4 into 2 and 3

($\alpha-\beta-\delta$) +β+δ=b

a-β-δ+β+δ=b

a+β+4δ=b

a+β+4δ=b…………5

a+2β+7δ=b…………6

-β-3δ=b-c

-β=b-c+3δ

B=-b+c-3δ

(a-β-δ)+3β+8δ=c

a-β-δ+3β+8δ=c

a+2β+7δ=c

sub β into 5

a+(-b+c-3δ)+4δ=b

a-b+c-3δ+4δ=b

a-b+c+δ=b

δ=b-a+b-c

δ=2b-a-c

Sub δ into β

B=-b+c-3(2b-a-c)

B=-b+c-6b+3a+3c

B=-7b+3a+4c

Sub β and 5 into 4

α=a-(-7b+3a+4c)-(2b-a-c)

=a+7b-3a-4c-2b+a+c

=-a+5b-3c

3. P=(1,2,3)

Q=(3,2,1)

R=(0,0,1)

SOLUTION

First check if its linear or not

$$\alpha \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

α+3β=0……..1

2α+2β=0…….2

3α+β+δ=0……3

From equation 1

α=-3β……..4

sub 4 into 2

2(-3β)+2β=0

-6β+2β=0

-4β=0

β=0

subβ into 4

α=-3(0)

α=0

sub α andβ into 3

3(0)+0+δ=0

0+0+δ=0

δ=0

α=0, β=0 and δ=0

hence the vectors are linearly dependent

Spanning Sets

$$\alpha \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

α+3β=0……..1

2α+2β=0…….2

3α+β+δ=0……3

From equation 1

$\alpha = a - 3\beta$………4

sub 4 into 2

$2(a-3\beta)+2\beta=b$

$2a-6\beta+2\beta=b$

$2a-4\beta = b$

$4\beta=2a-b$

$B=\frac{2a-b}{4}$

Sub β into 4

$A=a-3\left(\frac{2a-b}{4}\right)$

$\alpha=a-\frac{6a+3b}{4}$

$\alpha= \frac{4a-6a+3b}{4}$

$\alpha= \frac{-2a+3b}{4}$

Sub α and β into 3

$3\left(\frac{2a-b}{4}\right) + \left(\frac{2a-b}{4}\right) +\delta=c$

Multiply through by 4

$3(-2a+3b)+(2a-b)+4\delta=4c$

$-6a+9b+2a-b+4\delta=4c$

$-4a+8b+4\delta=4c$

Divide through by 4

$-a+2b+\delta=c$

$\delta=c+a-2b$

Since the vectors are linearly independent and spans of R3 sothe vectors are basis of R3