

JGIBORO JOSHUA Civil Engineering 19/ENG031013
MH104 ASSIGNMENT

$$1) y = \sin\left(\frac{6}{x}\right)$$

$$\text{let } u = \frac{6}{x} \quad (1)$$

$$y = \sin u$$

$$y + \Delta y = \sin(u + \Delta u)$$

$$\Delta y = \sin(u + \Delta u) - y$$

$$\Delta y = \sin(u + \Delta u) - \sin u$$

$$\Delta y = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\frac{\Delta y}{\Delta u} = \frac{2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\frac{\Delta u}{2}}$$

$$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\lim_{\Delta u \rightarrow 0} \left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \rightarrow 0} \left[\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \lim_{\Delta u \rightarrow 0} \left[\frac{\sin\left(\frac{\Delta u}{2}\right)}{\frac{\Delta u}{2}}\right]\right]$$

$$\therefore \frac{dy}{du} = \cos u$$

$$\text{from Equation (1): } u = \frac{6}{x}$$

$$u + \Delta u = \frac{6}{x + \Delta x} \Rightarrow u + \Delta u = \frac{-6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \rightarrow 0} \left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \rightarrow 0} \left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\frac{du}{dx} = -\frac{12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-1^2}{x^3}$$

$$\frac{dy}{dx} = \frac{-12 \cos u}{x^3}$$

Put the value of u back to eqn.

$$\therefore \frac{dy}{dx} = \frac{-12 \cos \left(\frac{6}{x^2}\right)}{x^3}$$

$$2) x = 4t^5 - t^3, y = t^4 + 2t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dy}{dt} = (12t^2 - 2t)dt$$

Let A rep the area

$$A = \int_a^b y \, dx$$

$$A = \int_1^3 (t^4 + 2t^2) (12t^2 - 2t) dt$$

$$\therefore A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3) dt$$

$$\Rightarrow \int_1^3 \frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 + C$$

$$\Rightarrow \left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81 \right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1 \right)$$

$$\Rightarrow \frac{160704}{35} - \frac{544}{105}$$

$$\therefore A = \underline{\underline{4586.36}} \text{ sq units}$$

$$3) x = 4t^5 - t^2, y = t^4 + 2t^2$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 - 4t$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$$

$$\therefore \frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$