MODEL OF A PERMANENT MAGNET SYNCHRONOUS MACHINE

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ABSTRACT
For the control system of a permanent magnet synchronous machine to be adequately designed the model and the correct parameters of the machine must be known.The aim of this write up is to discuss the conventional model of the permanent magnet synchronous machine**.**

INTRODUCTION

A permanent magnet synchronous machine comprises of phase windings in the stator and permanent magnets in the rotor. The stator carries a three-phase winding, which produces a near sinusoidal distribution of magneto motive force based on the value of the stator current. The magnets are mounted on the surface of the motor core.*(Dehkordi et al., 2005)*

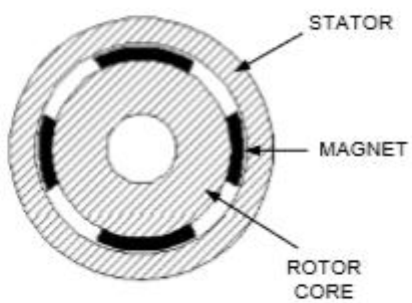

Figure 1: Structure of a permanent magnet synchronous machine *(Dehkordi et al., 2005)*

The permanent magnet in the PMSM performs the function of the field windings in the typical synchronous machine except that their magnetic field is constant and there is no control on it. The PMSM needs only AC supply for its operation unlike the typical synchronous machine which requires both AC and DC supply.*(Chapman, 2001)*

Mathematical model of PMSM

For proper simulation and analysis of the system, an accurate model is required. PMSMs used in industrial drives are normally closedloop controlled. The wye-connected three-phase PMSM has linearly dependent currents which cannot be used as independent state variables in the control design. Therefore, the three-phase PMSM model is transformed into a d-q-0 reference frame. The magnetically nonlinear characteristics of flux linkages are in such a way expressed as nonlinear functions of rotor position and the d-q reference frame currents.

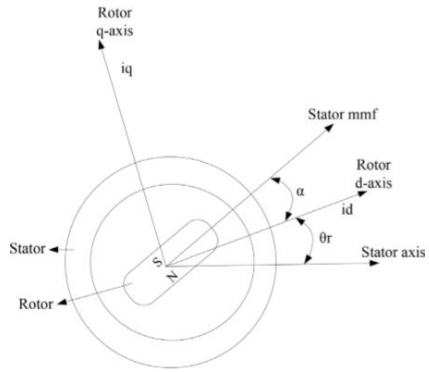

Figure 2: Motor axis *(Chapman, 2001)*

The motor axis has been developed using d-q rotor reference frame theory as shown in figure 2 above . At any particular time t, the rotor reference axis makes an angle θr with the fixed stator axis and the rotating stator mmf (magnetomotive force) creates an angle α with the rotor d axis. It is viewed that at any time t, the stator mmf rotates at the same speed as that of the rotor axis. *(Chapman, 2001)*

The stator of the PMSM and the wound rotor synchronous machine are similar. The permanent magnets used in the PMSM are of a modern rare-earth variety with high resistivity, so induced currents in the rotor are negligible. Also, there is no difference between the back EMF produced by a permanent magnet and that produced by an excited coil. Hence the mathematical model of a PMSM is similar to that of the wound rotor synchronous machine *(Pilla*y & Krishnan, 1989).To dererive the model the following assumptions are made:

1) Saturation is neglected although it can be taken into account by parameter changes.

2) The induced EMF is sinusoidal.

3) Eddy currents and hysteresis losses are negligible.

4) There are no field current dynamics.

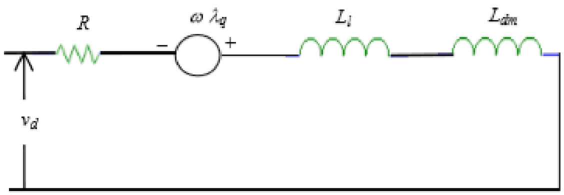

Figure 3: d axis model (Apte et al., 2015)

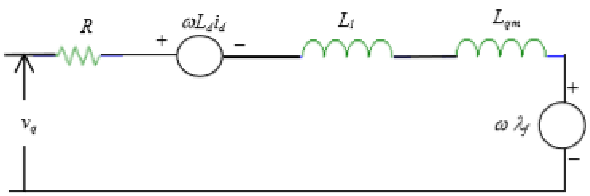

Figure 4: q axis model (Apte et al., 2015)

Following the assumptions, the stator d, q equations of the PMSM in the rotor reference frame are:

$$\boldsymbol{v_d = Ri_q + \rho\lambda_q + \omega_s\lambda_d} \qquad \textbf{(1)}$$

$$v_d = Ri_d + p\lambda_d - \omega_s\lambda_q \qquad \textbf{(2)}$$

where

$$\lambda_d = L_d i_d \qquad \textbf{(3)}$$

and

$$\lambda_d = L_d i_d + \lambda_{af} \qquad \textbf{(4)}$$

v_d and v_q are the d, q axis voltages, i_d and i_q are the d, q axis stator currents, L_d and L, are the d, q axis inductances, λ_d and λ_q, are the d, q axis stator flux linkages, while R and ω_s, are the stator resistance and inverter frequency, respectively. λ_{af} is the flux linkage due to the rotor magnets linking the stator.

The electric torque is given as:

$$T_e = 3P\left[\lambda_{af} i_q + \left(L_d - L_q\right) i_d i_q\right]/2 \quad \textbf{(5)}$$

and the equation for the motor dynamics is :

$$T_e = T_L + B\omega_r + J\rho\omega_r \quad \textbf{(6)}$$

P is the number of pole pairs, T_L is the load torque, B is the damping coefficient, ω_r , is the rotor speed, and J is the moment of inertia. The inverter frequency is related to the rotor speed as :

$$\omega_s = P\omega_r \quad \textbf{(7)}$$

The machine model is nonlinear because it contains product terms such as speed with i_d and i_q. Taking into consideration that that ω_r, i_q, and i_d are state variables.
For dynamic simulation, the equations of the PMSM presented in equ. (1)-(6) must be expressed in state-space form as shown in equ (8)-(10):

$$\rho i_d = (v_d - Ri_d + \omega_s L_q i_q)/L_d \quad \textbf{(8)}$$

$$\rho i_q = (v_q - Ri_q - \omega_s L_d i_d - \omega_s \lambda_{af})/L_q \quad \textbf{(9)}$$

$$\rho\omega_r = (T_e - T_L - B\omega_r)/J \quad \textbf{(10)}$$

The d, q variables are obtained from a, b, c variables through the Park transform defined below:

$$\begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} = 2/3 \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} . \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \textbf{(11)}$$

The a, b, c variables are obtained from the d, q variables using the inverse of the Park transform defined below:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \end{bmatrix} . \begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} \quad \textbf{(12)}$$

The Park transform converts time-varying differential equation into time-invariant differential equations.*(Mondal et al., 2014)*

The Park transform can also be applied to currents and flux linkages. The total input power to the machine in terms of the a, b,c variables is:

$$power = v_a i_a + v_b i_b + v_c i_c \quad \textbf{(13)}$$

while in d, q variables,

$$power = 3(v_d i_d + v_q i_q)/2 \quad \textbf{(14)}$$

for a balanced system..

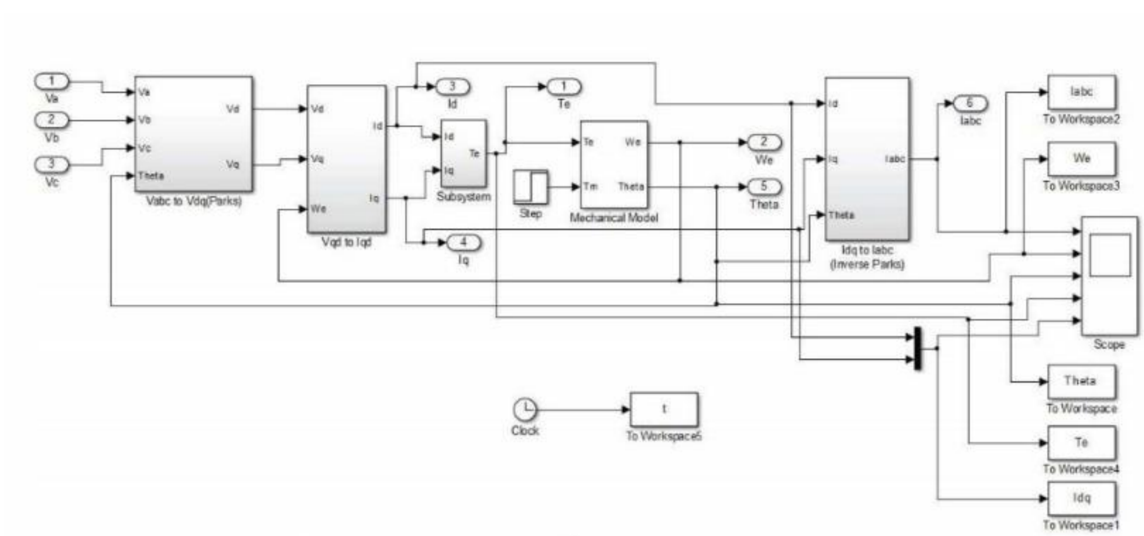

Figure 5: Simulink model of a PMSM Drive (Apte et al., 2015)

NOMENCLATURE

B	Damping constant N.m/rads/s.
v_a ,v_b ,v_c	a ,b and c phase voltage , V.
i_d , i_q	d and q axis stator currents .
J	Moment of inertia , kg.m^2 .
L_d ,L_q	stator d,q inductances, H .
ρ	Derivative operator .
P	Number of poles pairs .
R	Stator resistance ,Ω .
T_e	Electric torque , N.m .
T_L	Load torque, N.m.
v_d ,v_q	d and q axis stator voltages.
ω_r	Rotor speed, rad/s .
ω_s	Synchronous speed, rad/s.
λ_{af}	Mutual flux linkage between rotor and stator due to magnet, Wb.turn .
λ_d , λ_q	Stator d and q axis flux linkage, Wb.turn

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