JOHNSON VICTOR

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MAT 204

 LINEAR TRANSFORMATION

Linear transformation is a function from one vector space to another that respects the underlying (linear) structure of each vector space. A linear transformation is also known as a linear operator or map.

Examples of Linear Transformation

1. Verifying Linearity: dilation

Define T: R 2 → R 2 by T (x)= 1.5 x. Verify that T is linear.

T (u + v) = 1.5 (u + v) = 1.5 u + 1.5 v = T (u)+ T (v)

 T (cu)= 1.5 (cu)= c (1.5 u) = cT (u)

Since T satisfies both defining properties, T is linear

 $T\left(x\right)=\left(\begin{matrix}1.5&0\\0&1.5\end{matrix}\right)x$

 Since a matrix transformation is a linear transformation, this is another proof that T is linear.

1. Linearity Rotation

 Define T: R 2 → R 2 by T (x)= the vector x rotated counter clockwise by the angle θ. Verify that T is linear.

Since T is defined geometrically, we give a geometric argument. For the first property, T (u)+ T (v) is the sum of the vectors obtained by rotating u and v by θ . On the other side of the equation, T (u + v) is the vector obtained by rotating the sum of the vectors u and v. But it does not matter whether we sum or rotate first, as the following picture shows.



For the second property, cT (u) is the vector obtained by rotating u by the angle θ, then changing its length by a factor of c (reversing direction of c < 0. On the other hand, T (cu) first changes the length of c, then rotates. But it does not matter in which order we do these two operations.



This verifies that T is a linear transformation. We will find its matrix in the next subsection. Note however that it is not at all obvious that T can be expressed as multiplication by a matrix.

1. A transformation defined by Formula

Define T : R 2 → R 3 by the formula

 $T\left(\begin{matrix}x\\y\end{matrix}\right)=\left(\begin{matrix}3x-y\\y\\x\end{matrix}\right)$

We have to check the for *all* vectors u , v and *all* scalars c . In other words, we have to treat u , v , and c as *unknowns*; the only thing we are allowed to use is the definition of T . Since T is defined in terms of the coordinates of u , v , we need to give those names as well.

 $T\left(\left(\begin{matrix}x\_{1}\\y\_{1}\end{matrix}\right)+\left(\begin{matrix}x\_{2}\\y\_{2}\end{matrix}\right)\right)=T\left(\begin{matrix}x\_{1}+x\_{2}\\y\_{1}+y\_{2}\end{matrix}\right)=\left(\begin{matrix}3\left(x\_{1}+x\_{2}\right)-\left(y\_{1}+y\_{2}\right)\\y\_{1}+y\_{2}\\x\_{1}+x\_{2}\end{matrix}\right)$

 $=\left(\begin{matrix}3x\_{1}-y\_{1}\\y\_{1}\\x\_{1}\end{matrix}\right)+\left(\begin{matrix}3x\_{2}-y\_{2}\\y\_{2}\\x\_{2}\end{matrix}\right)=T\left(\begin{matrix}x\_{1}\\y\_{1}\end{matrix}\right)+T\left(\begin{matrix}x\_{2}\\y\_{2}\end{matrix}\right)$

 Hence T is in fact a matrix transformation.

1. A translation

Define T : R 3 → R 3 by

 $T\left(x\right)=x+\left(\begin{matrix}1\\2\\3\end{matrix}\right)$

This kind of transformation is called a *translation*. As in a previous example

, this T is not linear, because T ( 0 ) is not the zero vector.

1.

QUESTION 2

 M 1

 H 3

 I 5

 K 7

 O 9

 R

 C

 T(M)= 1

 T(H)= 3

 T(I)=5

 T(K)=7

 T(O)=9