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(43)

MATHS 104 MECHANICAL ENG

① $y = \sin\left(\frac{6}{x^2}\right)$

Let $u = 6/x^2$ --- ①

$\therefore y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u/2}$$

$$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \cdot \lim_{\Delta u \to 0}\left[\sin\left(\Delta u/2\right)\right]$$

$$\therefore \frac{dy}{du} = \cos u$$

From Equation (1); $u = 6/x^2$

$$u + \Delta u = \frac{6}{(x + \Delta x)^2}$$

$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0} \left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0} \left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\therefore \frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12 \cos u}{x^3}$$

Putting the value of u back

$$\therefore \frac{dy}{dx} = \frac{-12 \cos(6/x^2)}{x^3}$$

2) $x = 4t^3 - t^2, y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t, \; dx = (12t^2 - 2t)\,dt$

Let A represent the area

$A = \int_a^b y\,dx$

$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$

$\therefore A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$\Rightarrow \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4\right]_1^3 + C$

$\left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81\right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right)$

$= \frac{160704}{35} - \frac{544}{105}$

$\therefore A = 4586.36$ sq units

3) $x = 4t^3 - t^2, y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = 4t^3 - 4t$

$\frac{dx}{dt} = 12t^2 - 2t, \; \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$

$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2-1)}{2t(6t-1)}$$

$$\therefore \frac{dy}{dx} = \frac{2(t^2-1)}{6t-1}$$