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DEPT: COMPUTER ENGINEERING
COURSE: MAT 304

1. Differentiate $y = \sin(6/x^2)$ from the 1st principle

$$y + \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right)$$

$$y + \Delta y = \sin\left(\frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}\right)$$

$$\Delta y = \sin\left(\frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}\right) - y$$

$$\Delta y = \sin\left(\frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}\right) - \sin\left(\frac{6}{x^2}\right)$$

$$\Delta y = 2\cos\left[\frac{\left(\frac{6x^2 + 6(x+\Delta x)^2}{(x+\Delta x)^2(x^2)}\right)}{2}\right] \sin\left[\frac{\left(\frac{6x^2 - 6(x+\Delta x)^2}{(x+\Delta x)(x^2)}\right)}{2}\right]$$

$$\Delta y = 2\cos\left[\frac{6x^2 + 6x^2 + 12x\Delta x + 6(\Delta x)^2}{2x^4 + x^3\Delta x + 2x^2(\Delta x)^2}\right] \sin\left[\frac{-12x\Delta x - 6 - 6x^2 - 6x^2}{12x\Delta x - 6(\Delta x)^2}\right]$$

$$\Delta y = 2\cos\left[\frac{12x^2 + 12x\Delta x + 6(\Delta x)^2}{2x^4 + 4x^3\Delta x + 2x^2(\Delta x)^2}\right] \sin\left[\frac{-12x\Delta x - 6(\Delta x)^2}{2x^4 + 4x^3\Delta x + 2x^2(\Delta x)^2}\right]$$

$$\Delta y = 2\cos\left[\frac{6x^2 + 6x\Delta x + 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right] \sin\left[\frac{-6x\Delta x - 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right]$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{6x^2 + 6x\Delta x + 3(\Delta x)^2}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right] \sin\left[\frac{\Delta x(-6x - 3\Delta x)}{x^4 + 2x^3\Delta x + x^2(\Delta x)^2}\right]$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6x - 3(0)}{x^4 + 2x^3(0) + x^2(0)^2}\right)\cos\left(\frac{6x^2 + 6x(0) + 3(0)^2}{x^4 + 2x^3(0) + x^2(0)^2}\right)$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6x}{x^4}\right)\cos\left(\frac{6x^2}{x^4}\right)$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6}{x^3}\right)\cos\left(\frac{6}{x^2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{-2\cos(6/x^2)}{x^3}$$

2. Find the area under the curve given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^3$, at $t = 1$ and $t = 3$

$$A = \int_1^3 y\,dx$$

B.

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$A = \int_1^3 (12t^2 - 2t)(t^4 + 2t^3)\,dt$$

$$A = \int_1^3 12t^6 + 24t^5 - 2t^5 - 4t^3\,dt$$

$$A = \left[\frac{12t^7}{7} + \frac{24t^5}{5} - \frac{2t^6}{6} - \frac{4t^4}{4}\right]$$

$$A = \left[\frac{12t^7}{7} + \frac{24t^5}{5} - \frac{t^6}{3} - t^4\right]\Bigg|_1^3$$

$$A = \left[\frac{12(3)^7}{7} + \frac{24(3)^5}{5} - \frac{(3)^6}{3} - (3)^4\right] - \left[\frac{12(1)^7}{7} + \frac{24(1)^5}{5} - \frac{(1)^6}{3} - (1)^4\right]$$

$$A = \frac{26244}{7} + \frac{5832}{5} - 243 - 81 - \frac{12}{7} + \frac{24}{5} - \frac{1}{3} - 1$$

$$A = 4593.29 \text{ square unit}$$

3) If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$. Find dy/dx.

$$\frac{dx}{dt} = 12t^2 - 2t \qquad \frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$= \frac{2t(2t^2 + 2)}{2t(6t - 1)}$$

$$\frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}$$