

19 (ENG 061005)

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Mechanical Engineering

MAI 102

1) find a unit vector tangent to the space curve $x=t, y=t^2, z=t^3$ at the point where $t=1$

$$r'(t) = (1, 2t, 3t^2)$$

$$\|r'\| = \sqrt{(1)^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$$

$$\hat{r}(t) = \frac{r'(t)}{\|r'\|} = \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} (1, 2t, 3t^2)$$

$$= \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} (1, 2t, 3t^2)$$

$$= \frac{1}{\sqrt{1 + 4(1)^2 + 9(1)^4}} (1, 2(1), 3(1)^2) = \frac{1}{\sqrt{1 + 4 + 9}} (1, 2, 3)$$

$$= \frac{1}{\sqrt{14}} (1, 2, 3) = \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

2) $A = 4t^3 j + 5k, B = 2t^2 + 4t j$

$$G = A \times B = \begin{vmatrix} i & j & k \\ 0 & 4t^3 & 5 \\ 2t^2 & 4t & 0 \end{vmatrix}$$

$$= i \begin{vmatrix} 4t^3 & 5 \\ 4t & 0 \end{vmatrix} - j \begin{vmatrix} 0 & 5 \\ 2t^2 & 0 \end{vmatrix} + k \begin{vmatrix} 0 & 4t^3 \\ 2t^2 & 4t \end{vmatrix}$$

$$G = -20ti + 10t^2j - 8t^5k$$

$$\int G = \int (-20ti + 10t^2j - 8t^5k) dt$$

$$= -10t^2i + \frac{10}{3}t^3j - \frac{8 \cdot 4}{3}t^6k$$

$$\int_0^1 G = \left[\left(-10(1)^2i + \frac{10}{3}(1)^3j - 4(1)^6k \right) - \left(-10(0)^2i + \frac{10}{3}(0)^3j - 4(0)^6k \right) \right]$$

$$= -10i + \frac{10}{3}j - \frac{4}{3}k$$