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Civil Engineering
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MAT 104

1 $y = \sin\left(\frac{6}{x^2}\right)$

let $u = 6/x^2$

$\therefore y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u+\Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \times \sin\left(\frac{\Delta u}{2}\right)$

$\frac{\Delta y}{\Delta u} = 2\cos\left(\frac{2u + \Delta u}{2}\right) \times \sin\left(\frac{\Delta u}{2}\right) \times 1/2$

$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \times \frac{\sin(\Delta u/2)}{\Delta u/2}$

$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \times \lim_{\Delta u \to 0}\left[\frac{\sin(\Delta u/2)}{\Delta u/2}\right]$

$\therefore dy/du = \cos u$

~~Since~~

Since $u = 6/x^2$

$u + \Delta u = \frac{6}{(x+\Delta x)^2}$

$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - u$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) + (\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \therefore \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\therefore \frac{du}{dx} = -\frac{12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times -\frac{12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$$

Where $u = 6/x^2$

$$\therefore \frac{dy}{dx} = \frac{-12\cos(6/x^2)}{x^3}$$

2 $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$, $dx = (12t^2 - 2t)dt$

Let the Area be A

$$A = \int_a^b y\,dx$$

$$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$$

$$A = \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4\right]_1^3$$

$$= \left(\frac{12(3)^7}{7} - \frac{(3)^6}{3} + \frac{24(3)^5}{5} - (3)^4\right) - \left(\frac{12(1)^7}{7} - \frac{(1)^6}{3} + \frac{24(1)^5}{5} - (1)\right)$$

$$= \left(\frac{26244}{7} - \frac{243}{3} + \frac{5832}{5} - 81\right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right)$$

$$= \frac{160704}{35} - \frac{544}{105}$$

$\therefore A = 4586.36$ square Unit

2) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$dy/dx = dy/dt \times dt/dx$

$dy/dx = 4t^3 - 4t$

$dx/dt = 12t^2 - 2t$

$\therefore dt/dx = \dfrac{1}{12t^2 - 2t}$

$dy/dx = 4t^3 - 4t \times \dfrac{1}{12t^2 - 2t}$

$dy/dx = \dfrac{4t^3 - 4t}{12t^2 - 2t}$

$= \dfrac{4t(t^2 - 1)}{2t(6t - 1)}$

$\therefore \dfrac{dy}{dx} = \dfrac{2(t^2 - 1)}{6t - 1}$