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Matric Number: 19/ENG04/012
Serial No: 152

MATH104.

(1) Soln: diff $y = \sin\left(\frac{6}{x^2}\right)$ from the first Principle.

$\therefore$ Applying Chain Rule: $\frac{d}{dx}(f(g)) = \frac{d}{dg}(f(g)) \times \frac{d}{dx}(g)$

where $g = \frac{6}{x}$

$\therefore \frac{d}{dx}\sin\left[\frac{6}{x^2}\right] = \frac{d}{dg}(\sin(g)) \times \frac{d}{dx}\left[\frac{6}{x^2}\right]$

Solve $\frac{d}{dx}\left[\frac{6}{x^2}\right]$ Using quotient Rule. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$v = x^2,\ u = 6,\ \frac{du}{dx} = 0,\ \frac{dv}{dx} = 2x$

$= \frac{x^2 - 6(2x)}{(x^2)^2} = \frac{x^2 - 12x}{x^4} =$

(1) $y = \sin(6/x^2)$

Soln $y + \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right)$

$\therefore \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right) - \sin\left(\frac{6}{x^2}\right)$

$\therefore \sin A - \sin B = \left[2\cos\frac{A+B}{2}\ \sin\frac{A-B}{2}\right]$

$(x+\Delta x)^2 = x^2 + 2x\Delta x + \Delta x^2$

$\therefore \Delta y = 2\cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2(x+\Delta x)^2(x^2)}\right] \cdot \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2(x+\Delta x)^2 x^2}\right]$

$\therefore \frac{\Delta y}{\Delta x} = 2\cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x(x+\Delta x)^2}\right] \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right] \times -\frac{6}{x^3}$

$\frac{\Delta y}{\Delta x} = \frac{-12\cos}{x^3}\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right] \cdot \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2x^2[x+\Delta x]^2}\right]$

lim $\Delta x \to 0$ lim $\to \Delta x \to 0$ $6\Delta 4$ lim $\Delta x \to 0$

$\frac{\Delta y}{\Delta x} = -12\cos\left[\frac{12x^2}{2x^2[x]^2}\right] \cdot 1$ $\therefore \frac{dy}{dx} = \frac{-12\cos}{x^3}\left(\frac{12x^2}{2x^4}\right) = \frac{-12\cos}{x^3}\left[\frac{6}{x^2}\right]$

$= \frac{-12\cos(6/x^2)}{x^3}$

(2)

$x = 4t^3 - t^2$ $y = t^4 + 2t^2$

A = area

$$A = \int_a^b y\,dx$$

$\therefore y = t^4 + 2t^2$ then $A = \int_a^b t^4 + 2t^2\,dx$

Given $x = 4t^3 - t^2$ $\therefore dx/dt = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$$\therefore A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$$

$$\therefore A = \int_1^3 (12t^6 + 2t^5 + 24t^4 - 4t^3)\,dt$$

$$\therefore A = \left[\frac{12t^7}{7} + \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}\right]_1^3$$

$$\therefore A = \left[\frac{12(3)^7}{7} + \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4}\right] - \left[\frac{12(1)^7}{7} + \frac{2(1)^6}{6} + \frac{24(1)^5}{5} + \frac{4(1)^4}{4}\right]$$

$$\frac{160704}{35} - \frac{544}{105}$$

$= 4586.36$ square units.

(3)

$x = 4t^3 - t^2$ $y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$ $\frac{dy}{dt} = 4t^3 + 4t$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4t^3 + 4t}{12t^2 - 2t} = \frac{4t(t^2 + 1)}{2t(6t - 1)} =$$

$$= \frac{2t(t^2 + 1)}{(6t - 1)}$$

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