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17/MHS01/083
Civil Engineering.

① $y = \sin\left(\frac{6}{x^2}\right)$

let $u = 6/x^2$ --- ①

$\therefore y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u/2}$$

$$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \frac{\sin\left(\frac{\Delta u}{2}\right)}{\frac{\Delta u}{2}}$$

$$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \cdot \lim_{\Delta u \to 0}\left[\frac{\sin\left(\Delta u/2\right)}{\Delta u/2}\right]$$

$$\therefore \frac{dy}{du} = \cos u$$

From equation 1 $u = 6/x^2$

$$u + \Delta u = \frac{6}{(x+\Delta x)^2}$$

$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\therefore \frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$dy/dx = \frac{-12\cos u}{x^3}$$

putting the value of u back

$$\therefore dy/dx = \frac{-12\cos(6/x^2)}{x^3}$$

2 $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$, $dx = (12t^2 - 2t)dt$

let A represent the area

$A = \int_a^b y\,dx$

$A \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$

$\therefore A = \int_1^3 (12t^6 - 2t^5 + 24 + t^4 - 4t^3)\,dt$

$= \Big|_1^3 \frac{12t^4}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 + c$

$\left[\frac{26244}{7} - \frac{243 + 5832}{5} - 81\right] - \left[\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right]$

$= \frac{160704}{35} - \frac{544}{105}$

$\therefore A = 4586.36$ sq units

3. $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dt}{dx}$

$\frac{dy}{dx} = 4t^3 - 4t$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$$

$$\therefore \frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$