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Electrical Electronics Engineering
Maths 104

$$1) y = \sin\left(\frac{6}{x^2}\right)$$

$$\text{Let } u = \frac{6}{x^2} \dots \textcircled{1}$$

$$y = \sin u$$

$$y + \Delta y = \sin(u + \Delta u)$$

$$\Delta y = \sin(u + \Delta u) - y$$

$$\Delta y = \sin(u + \Delta u) - \sin u$$

$$\Delta y = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) / \frac{\Delta u}{2}$$

$$\frac{\Delta y}{\Delta u} = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) / \frac{\Delta u}{2}$$

$$\lim_{\Delta u \rightarrow 0} \left(\frac{\Delta y}{\Delta u} \right) = \lim_{\Delta u \rightarrow 0} \left[\cos\left(\frac{2u + \Delta u}{2}\right) \right] \cdot \lim_{\Delta u} \left[\frac{\sin(\Delta u/2)}{\Delta u/2} \right] \quad 2)$$

$$dy/du = \cos u$$

From equation (1), $u = \frac{6}{x^2}$

$$u + \Delta u = \frac{6}{(x + \Delta x)^2}$$

$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = -12x(\Delta x) - 6(\Delta x)$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \rightarrow 0} \left(\frac{\Delta u}{\Delta x} \right) = \lim_{\Delta x \rightarrow 0} \left[\frac{-12x - 6(\Delta x)}{x^4} \right]$$

$$\frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12 \cos u}{x^3}$$

Putting the value of u back.

$$\frac{dy}{dx} = \frac{-12 \cos\left(\frac{6}{x^2}\right)}{x^3}$$

2) $x = 4t^3 - t^2, y = t^4 + 2t^2$

$$\frac{dx}{dt} = 12t^2 - 2t, dx = (12t^2 - 2t) dt$$

Let A represent the area.

$$A = \int y dx$$

$$A = \int_1^3 (t^4 + 2t^2) (12t^2 - 2t) dt$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3) dt$$

$$= \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - t^4 + C \right]_1^3$$

$$\left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81 \right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1 \right)$$

$$= \frac{160704}{35} - \frac{544}{105}$$

$$A = 4586.36 \text{ sq units}$$

$$3) \quad x = 4t^3 - t^2, \quad y = t^4 + 2t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 - 4t$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$$
$$\frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$