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(1) $\int e^x \sin x \, dx$

Solution

$$u = \sin x \quad dv = e^x$$

$$\frac{du}{dx} = \cos x = e^x$$

$$du = \cos x \, dx$$

$$\int u \, dv = uv - \int v \, du$$

$$= \sin x \cdot e^x - \int e^x \cos x \, dx$$

$$= \sin x \, e^x -$$

$$\int e^x \cos x \, dx$$

$$u = \cos x \quad dv = e^x$$

$$du/dx = -\sin x$$

$$v = e^x$$

$$du = -\sin x \, dx$$

$$\int u \, dv = uv - \int v \, du$$

$$= e^x \cos x - \int e^x (-\sin x) \, dx$$

$$\begin{aligned}
 &= \sin x e^x - e^x \cos x - \int e^x (-\sin x) dx \\
 &= e^x \sin x (e^x \cos x + \int e^x \sin x dx) \\
 &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \\
 &= \int e^x \sin x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx \\
 &\text{let } p = \int e^x \sin x dx
 \end{aligned}$$

$$p = e^x \sin x - e^x \cos x - p$$

collect like terms

$$2p = e^x \sin x - e^x \cos x$$

$$p = \frac{e^x \sin x - e^x \cos x}{2}$$

2

therefore

$$\int e^x \sin x dx = \frac{e^x \sin x - e^x \cos x}{2}$$

2

$$\int e^x \sin x dx = \frac{1}{2} (e^x \sin x - e^x \cos x) + c$$

$$(2) \int 2x^2 \ln x dx$$

Solution

$$\ln x = \log_e x$$

$$\int 2x^2 \log_e x dx$$

$$u = \log_e x, \quad du = \frac{1}{x} dx$$

$$v = \frac{2x^3}{3}$$

$$\frac{dv}{dx} = \frac{2}{3} x^2$$

$$dv = \frac{2}{3} x^2 dx$$

$$\int u dv = uv - \int v du$$

$$\frac{2x^3}{3} - \log_e x - \int \frac{2x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{2x^3}{3} \log_e x - \int \frac{2x^2}{3} dx$$

$$\frac{2x^3}{3} \log x - \frac{2x^3}{9} + c_4$$

~~$$(3) x^2$$~~

$$(3) x^2 \sin x dx$$

Solution,

$$u = x^2 : du = 2x dx$$

$$\frac{du}{dx} = 2x \quad \& \quad v = -\cos x$$

$$du = 2x dx$$

$$\int u dv = uv - \int v du$$

$$= x^2 (-\cos x) - \int (-\cos x) \cdot 2x dx$$

$$= -x^2 \cos x + \int \cos x \cdot 2x dx$$

$$\int \cos x \cdot 2x dx$$

$$u = 2x \quad du = 2 dx$$

$$du/dx = 2$$

$$v = \sin x$$

$$du = 2 dx$$

$$\begin{aligned}
 \int u dv &= uv - \int v du \\
 &= 2x \cdot \sin x - \int \sin x \cdot 2 \\
 &= 2x \sin x - (-2 \cos x) \\
 &= 2x \sin x + 2 \cos x
 \end{aligned}$$

Here

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + c$$

4. $\int x \cos x dx$

solution

$$\begin{aligned}
 u &= x & dv &= \cos x \\
 \frac{du}{dx} &= 1 & v &= \sin x \\
 \frac{du}{dx} &= 1
 \end{aligned}$$

$$\begin{aligned}
 \int u dv &= uv - \int v du \\
 &= x \cdot \sin x - \int \sin x dx \\
 &= x \sin x - (-\cos x) \\
 &= x \sin x + \cos x + c
 \end{aligned}$$