Henshaw David .G.
Civil Engineering
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MAT 104 Assignment

① $y = \sin\left(\frac{6}{x^2}\right)$

let $u = \frac{6}{x^2}$

$y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \times \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\frac{\Delta u}{2}}$

$\frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \times \sin\left(\frac{\Delta u}{2}\right) \div \frac{\Delta u}{2}$

$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0} \cos\left(\frac{2u + \Delta u}{2}\right) \times \lim_{\Delta u \to 0} \frac{\sin\left(\frac{\Delta u}{2}\right)}{\frac{\Delta u}{2}}$

$\frac{dy}{du} = \cos u$

recall $u = \frac{6}{x^2}$

$\therefore u + \Delta u = \frac{6}{(x + \Delta x)^2}$

$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$

$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$

$\Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$

$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$

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$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{-12x - 6(\Delta x)^2}{x^4}\right)$$

$$\frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$$

recall $u = \frac{6}{x^2}$

$$\frac{dy}{dx} = \frac{-12\cos\left(\frac{6}{x^2}\right)}{x^3}$$

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2) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$, $\frac{dy}{dt} = 4t^3 + 4t$

$\therefore dx = (12t^2 - 2t)dt$

Let A be the Area

$A = \int_a^b y\,dx$

$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)dt$

$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)dt$

$\left| \frac{12t^7}{7} - \frac{2t^6}{63} + \frac{24t^5}{5} - t^4 + C \right|_1^3$

Solving mathematically

$A = 4586.36$ sq. units

3) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dt} = 4t^3 - 4t$

$\frac{dx}{dt} = 12t^2 - 2t$

$\frac{dy}{dx} = (4t^3 - 4t) \times \frac{1}{12t^2 - 2t}$

$\therefore \frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t}$

$= \frac{4t(t^2 - 1)}{2t(6t - 1)}$

$\therefore \frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$

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