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MAT 102

1. $x = t, y = t^2, z = t^3$ at $t = 1$. Find unit vector tangent to the curve

Sol

$$\vec{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \quad t = 1$$

$$\frac{d\vec{r}}{dt} = 2t\mathbf{j} + 3t^2\mathbf{k}$$

at $t = 1$

Tangent vector, $\frac{d\vec{r}}{dt} = 2(1)\mathbf{j} + 3(1)^2\mathbf{k}$

$$\frac{d\vec{r}}{dt} = 2\mathbf{j} + 3\mathbf{k}$$

$$\left| \frac{d\vec{r}}{dt} \right| = \sqrt{(2)^2 + (3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{14}$$

$$\mathbf{T} = \frac{\frac{d\vec{r}}{dt}}{\left| \frac{d\vec{r}}{dt} \right|} = \frac{2\mathbf{j} + 3\mathbf{k}}{\sqrt{14}}$$

2. $A = t^3\mathbf{j} + 5\mathbf{k}, B = 2t^2\mathbf{i} + 4t\mathbf{j}$. $\mathbf{C} = A \times B$. Find integral of \mathbf{C} from (0 to 1)

$$\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4t^3 & 5 \\ 2t^2 & 4 & 0 \end{vmatrix}$$

$$\mathbf{i} \begin{vmatrix} 4t^3 & 5 \\ 4 & 0 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 0 & 5 \\ 2t^2 & 0 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 0 & 4t^3 \\ 2t^2 & 4 \end{vmatrix}$$

$$\mathbf{i}(0 - 20) - \mathbf{j}(0 - 10t^2) + \mathbf{k}(0 - 8t^5)$$

$$\mathbf{C} = -20\mathbf{i} + 10t^2\mathbf{j} - 8t^5\mathbf{k}$$

$$\int_0^1 t \, dt = \frac{-20t}{1} + \frac{10t^3}{3} j - \frac{8t^6}{6} k$$

$$\left[\frac{-20(1)}{1} i + \frac{10(1)^3}{3} j - \frac{8(1)^6}{6} k \right] - \left[\frac{-20(0)}{1} i + \frac{10(0)^3}{3} j - \frac{8(0)^6}{6} k \right]$$

$$= -20i + \frac{10}{3} j - \frac{4}{3} k$$