Name : Ohwuchi Avitus Chukwuebuka
Department : Chemical Engineering
Matric No : 19/EN601/010

MAT 104

3 If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, Find $dy/dx$

$\frac{dy}{dt} = 4t^3 + 4t \qquad \frac{dx}{dt} = 12t^2 - 2t$

$\therefore \frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$

$= \frac{4t(t^2+1)}{2t(6t-1)}$

$\therefore \frac{dy}{dx} = \frac{2(t^2+1)}{6t-1}$

2 $x = 4t^3 - t^2$, $y = t^4 + 2t^3$, Find area under the curve at $t = 1$ and $t = 3$

SOL

Let A represent the area,

$A = \int_a^b$

$y = t^4 + 2t^2$

$A = \int_a^b t^4 + 2t^2 \, dx$

$x = 4t^3 - t^2$

$dx/dt = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$A = \int_1^3 t^4 + 2t^2 (12t^2 - 2t\,dt)$

$A = \int_1^3 12t^6 + 24t^4 - 4t^3 dt - 2t^5 dt$

$A = 12t^6 + 24t^4 \int(-4t^3 - 2t^5)\,dt$

$= \left[12t^6 + 24t^4 - t^4 - \frac{4}{5}t^6\right]_1^3$

$$= \left[12(3)^6 + 24(3)^4 - (3)^4 - \frac{1(3)^6}{5}\right] - \left[12(1)^6 + 24(1)^4 - (1)^4 - \frac{1(1)^6}{5}\right]$$

$$= [8748 + 1944 - 81 - 145.8] - [12 + 24 - 1 - 0.2]$$

$$\cancel{= 5751 - 33.67} \qquad \cancel{= 5751 - \frac{101}{3}} \qquad = 10,465.2 - 34.8$$

$$\cancel{= 5} \qquad \cancel{= 5717\tfrac{1}{3}} \qquad = 10,430.4$$

1 Differentiate $y = \sin(6/x^2)$ from first principle

$$y + \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right)$$

$$y + \Delta y = \sin\left(\frac{6}{x^2 + 2x\Delta x + \Delta x^2}\right)$$

$$\Delta y = \sin\left(\frac{6}{x^2 + 2x\Delta x + \Delta x^2}\right) - \sin\left(\frac{6}{x^2}\right)$$

Using factor formula

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$