(3) $\int x^2 \sin x \, dx$

Solution

$u = x^2 \qquad dv = \sin x$

$du = 2x\,dx \qquad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2(-\cos x) - \int (-\cos x) \times 2x\,dx$

$= -x^2 \cos x + \int 2x \cos x \, dx$

$\int 2x \cos x \, dx$

$u = 2x \qquad dv = \cos x$

$du = 2\,dx \qquad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= 2x \sin x - \int \sin x \times 2\,dx$

$= 2x \sin x - 2\int \sin x \, dx$

$= 2x \sin x - 2(-\cos x)$

$= 2x \sin x + 2\cos x$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

2

(4) $\int x \cos x \, dx$

Solution

$u = x \qquad dv = \cos x$

$du = 1\,dx \qquad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= x \sin x - \int \sin x \times 1\,dx$

$= x \sin x - \int \sin x \, dx$

$= x \sin x - (-\cos x) + C$

$= x \sin x + \cos x + C$

4

ASSIGNMENT

1. $\int e^{x}\sin x\, dx$

Solution

$u = \sin x \qquad dv = e^{x}$

$du = \cos x\, dx \qquad v = e^{x}$

$\int u\,dv = uv - \int v\,du$

$= (\sin x)(e^{x}) - \int e^{x}\cos x\, dx$

$= e^{x}\sin x - \int e^{x}\cos x\, dx$

$\int e^{x}\cos x\, dx$

$u = \cos x \qquad dv = e^{x}$

$du = -\sin x\, dx \qquad v = e^{x}$

$\int u\,dv = uv - \int v\,du$

$= e^{x}\cos x - \int e^{x}(-\sin x\, dx)$

$= e^{x}\cos x + \int e^{x}\sin x\, dx$

$\int e^{x}\sin x\, dx = e^{x}\sin x - \left[e^{x}\cos x + \int e^{x}\sin x\, dx\right]$

$\int e^{x}\sin x\, dx = e^{x}\sin x - e^{x}\cos x - \int e^{x}\sin x\, dx$

Let $\int e^{x}\sin x\, dx = I$

$I = e^{x}\sin x - e^{x}\cos x - I$

$I + I = e^{x}\sin x - e^{x}\cos x$

$\frac{\cancel{2}I}{\cancel{2}} = \frac{e^{x}\sin x}{2} - \frac{e^{x}\cos x}{2}$

$I = \frac{e^{x}\sin x}{2} - \frac{e^{x}\cos x}{2}$

$\int e^{x}\sin x\, dx = \frac{e^{x}\sin x - e^{x}\cos x}{2} + c$

$\int e^{x}\sin x\, dx = 1/2\left[e^{x}\sin x - e^{x}\cos x\right] + c$

(2) $\int 2x^2 \ln x \, dx$

Solution

$2\int x^2 \ln x$

$u = \ln x \qquad dv = x^2$

$du = 1/x \, dx \qquad v = x^3/3$

$\int u\,dv = uv - \int v\,du$

$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \times \frac{1}{x} \, dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} \, dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2 \, dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3} \times \frac{x^3}{3}\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right]$

$\therefore \int 2x^2 \ln x \, dx = \left[\frac{2 \cdot x^3}{3}\left(\frac{\ln(x)}{1} - \frac{1}{3}\right)\right] + C$