OKE-OLUO TIMOTHY
19/ENG06/045
Mechanical

Mat 104

$y = \sin\left(\frac{6}{x^2}\right)$

Let $y = \sin(u)$

$\frac{dy}{du} = \cos(u)$

$u = \frac{6}{x^2}$

$u + \Delta u = \frac{6}{(x+\Delta x)^2} = \frac{6}{x^2 + 2x\Delta x + (\Delta x)^2}$

$\Delta u = \frac{6}{x^2 + 2x\Delta x + \Delta x^2} - \frac{6}{x^2}$

$= \frac{6x^2 - 6(x^2 + 2x\Delta x + \Delta x^2)}{x^2(x^2 + 2x\Delta x + \Delta x^2)}$

$= \frac{6x^2 - 6x^2 - 12x\Delta x - 6\Delta x^2}{x^4 + 2x^3\Delta x + x^2\Delta x^2} = \frac{\Delta x(-12x - 6\Delta x)}{x^4 + 2x^3\Delta x + x^2\Delta x^2}$

$\frac{\Delta u}{\Delta x} = \frac{\Delta x(-12x - 6\Delta x)}{x^4 + 2x^3\Delta x + x^2\Delta x^2} \div \Delta x$

$= \frac{\Delta x(-12x - 6\Delta x)}{x^4 + 2x^3\Delta x + x^2\Delta x^2} \times \frac{1}{\Delta x}$

$= \frac{-12x - 6\Delta x}{x^4 + 2x^2\Delta x + x^2\Delta x^2}$

$\lim_{\Delta x \to 0} \frac{\Delta}{\Delta x} = \frac{\lim_{\Delta x \to 0}(-12x) - \lim_{\Delta x \to 0}(6\Delta x)}{\lim_{\Delta x \to 0}(x^4) + \lim_{\Delta x \to 0}(2x^2\Delta x) + \lim_{\Delta x \to 0}(x^2\Delta x^2)}$

$\frac{dy}{dx} = \frac{-12x}{x^4} = 12x^{-3}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \cos u \, x - 12x^{-3}$

$= -12x^{-3}\cos(6/x^2)$

$= -\frac{12}{x^3}\cos(6/x^2)$

2) $x = 4t^3 - t^2$

$y = 4t + 2t^2$

Sit $t = 1$ and $t = 3$

Given $y = t^4 + 2t^2$

$$A = \int_1^3 t^2 + 2t^2 \, dx$$

Given $x = 4t^3 - t^2$

$$\frac{dx}{dt} = 12t - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$A = \int_1^3 (4t^3 - t^2)(12t^2 - 2t)\,dt$$

$$A = \int_1^3 [48t^5 - 20t^4 + 2t^3]\,dt$$

$$A = [8t^6 - 4t^5 + \tfrac{1}{2}t^4]_1^3$$

$$A = [8t^6 - 4t^5 + \tfrac{1}{2}t^4]_1^3$$

$$A = [8(3)^6 - 4(3)^5 + \tfrac{1}{2}(3)^4] - [8(1)^6 - 4(1)^5 + \tfrac{1}{2}(1)^4]$$

$$A = [8(729) - 4(243) + \tfrac{1}{2}(81)] - [8(1) - 4(1) + \tfrac{1}{2}(1)]$$

$$A = \left[5832 - 972 + \frac{81}{2}\right] - \left[8 - 4 + \frac{1}{2}\right]$$

$$A = \frac{9801}{2} - \frac{9}{2}$$

$A = 4896$ square units

3) $x = 4t^3 + t^2$

$y = t^4 + 2t^2$

$$\frac{dx}{dt} = 12t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4t^3 + 4t}{12t^2 - 2t}$$