

MAT 204

COMPUTER SCIENCE

A **linear transformation**, $T:U \rightarrow V$, is a function that carries elements of the vector space U (called the **domain**) to the vector space V (called the **codomain**), and which has two additional properties

1. $T(u_1+u_2)=T(u_1)+T(u_2)$ for all $u_1, u_2 \in U$
2. $T(\alpha u)=\alpha T(u)$ for all $u \in U$ and all $\alpha \in \mathbb{C}$

$$A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 1 & -1 \\ 5 & 2 & -4 \end{pmatrix}$$
$$D = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{pmatrix} \quad E = \begin{pmatrix} 3 & 3 & 0 \\ 2 & 1 & -1 \\ 1 & 3 & -1 \end{pmatrix}$$

i. **LINEAR TRANSFORMATION OF A, IF VEXTOR $x = (a,b,c)$**

$$A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix} \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} + b \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ -2b \end{pmatrix} + \begin{pmatrix} 3c \\ 2c \\ c \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 0 + b + 3c \\ -a + 0 + 2c \\ 4a - 2b + c \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} 0 + b + 3c \\ -a + 0 + 2c \\ 4a - 2b + c \end{pmatrix}$

ii. **LINEAR TRANSFORMATION OF B , IF VEXTOR x = (a,b,c)**

$$B = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 0 \\ -a \\ 4a \end{pmatrix} + b \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4a \\ 3a \\ 2a \end{pmatrix} + \begin{pmatrix} 2b \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} c \\ -c \\ c \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 4a + 2b + c \\ 3a + b - c \\ 2a + 0 + c \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} 4a + 2b + c \\ 3a + b - c \\ 2a + c \end{pmatrix}$

iii. **LINEAR TRANSFORMATION OF C , IF VEXTOR $x = (a,b,c)$**

$$C = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 1 & -1 \\ 5 & 2 & -4 \end{pmatrix} \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} + b \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + c \begin{pmatrix} 0 \\ -1 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 4a \\ 6a \\ 5a \end{pmatrix} + \begin{pmatrix} 3b \\ b \\ 2b \end{pmatrix} + \begin{pmatrix} 0 \\ -c \\ -4c \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 4a + 3b \\ 6a + b - c \\ 5a + 2b - 4c \end{pmatrix}$$

Hence the transformation.

Gives

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \begin{pmatrix} 4a + 3b + 0 \\ 6a + b - c \\ 5a + 2b - 4c \end{pmatrix}$$

iv. **LINEAR TRANSFORMATION OF D , IF VEXTOR $x = (a,b,c)$**

$$D = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{pmatrix} \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} + b \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + c \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ a \\ 4a \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 2b \end{pmatrix} + \begin{pmatrix} 3c \\ 2c \\ c \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 0 + b + 3c \\ a + 0 + 2c \\ 4a + 2b + c \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} 0 + b + 3c \\ a + 0 + 2c \\ 4a + 2b + c \end{pmatrix}$

v. LINEAR TRANSFORMATION OF E , IF VEXTOR x = (a,b,c)

$$E = \begin{pmatrix} 3 & 3 & 0 \\ 2 & 1 & -1 \\ 1 & 3 & -1 \end{pmatrix} \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + b \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + c \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 3a \\ 2a \\ a \end{pmatrix} + \begin{pmatrix} 3b \\ b \\ 3b \end{pmatrix} + \begin{pmatrix} 0 \\ -c \\ -c \end{pmatrix}$$

$$\begin{pmatrix} 3a + 3b + 0 \\ 2a + b - c \\ a - 3b - c \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} 3a + 3b + 0 \\ 2a + b - c \\ a - 3b - c \end{pmatrix}$