$$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\therefore \frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$$

Putting the value of u back

$$\frac{dy}{dx} = \frac{-12\cos(6/x^2)}{x^3}$$

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1) $y = \sin\left(\frac{6}{x^2}\right)$

Let $u = 6/x^2$ — (i)

$y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u / 2}$$

$$\frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \frac{\sin\left(\frac{\Delta u}{2}\right)}{\Delta u / 2}$$

$$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\frac{(2u + \Delta u)}{2}\right] \cdot \lim_{\Delta u \to 0}\left[\frac{\sin(\Delta u/2)}{\Delta u/2}\right]$$

$\frac{dy}{du} = \cos u$

from equation (i); $u = 6/x^2$

$$u + \Delta u = \frac{6}{(x + \Delta x)^2}$$

2) $x = 4t^3 - t^2,\ y = t^4 + 2t^3$

$\frac{dx}{dt} = 12t^2 - 2t,\ dx = (12t^2 - 2t)\,dt$

Let A represent the area

$A = \int_{1}^{3} y\,dx$

$A = \int_{1}^{3} (t^4 + 2t^3)(12t^2 - 2t)\,dt$

$A = \int_{1}^{3} (12t^6 - 2t^5 + 24t^4 - 4t^2)\,dt$

$\Rightarrow \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 + c\right]_1^3$

$\left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81\right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right)$

$= \frac{160704}{35} - \frac{548}{105}$

$A = 4586.36$ sq units

3.) $x = 4t^3 - t^2,\ y = t^4 + 2t^3$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = 4t^3 - 4t$

$\frac{dx}{dt} = 12t^2 - 2t,\quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$

$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$

$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$

$\frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$