1) Differentiat $y = \sin\left(\frac{6}{x^2}\right)$ from first principal

Solution

$y = \sin(6x^{-2})$

$y + \Delta y = \sin 6(x + \Delta x)^{-2}$

$y + \Delta y = \sin(6x^{-2} + 6\Delta x^{-2})$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - \sin 6x^{-2}$

$2\cos\frac{(A+B)}{2} \cdot \sin\frac{(A-B)}{2}$

$2\cos\frac{(6x^{-2} + 6\Delta x^{-2} + 6x^{-2})}{2} \sin\frac{(6x^{-2} + 6\Delta x^{-2} - 6x^{-2})}{2}$

$= 2\cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \sin\frac{(6\Delta x^{-2})}{2}$

divide both sides by Δx

$\frac{\Delta y}{\Delta x} = \frac{2\cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \sin\frac{(6\Delta x^{-2})}{2}}{\Delta x}$

$= \cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \sin\frac{(6\Delta x^{-2})}{2} \cdot \frac{1}{\frac{\Delta x}{2}}$

$\lim_{\Delta \to 0} = \cos\frac{(12x^{-2} + 6(0)^{-2})}{2} \sin\frac{6\Delta x^{-2}}{2} \cdot \frac{1}{\frac{\Delta x}{2}} \longrightarrow 1$

$\frac{dy}{dx} = \cos\frac{12x^{-2}}{2}$

$\frac{\Delta y}{\Delta x} = \cos 6x^{-2}$ //

2) Find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, at $t = 1$ and $t = 3$

Solution

given $y = t^4 + 2t^2$

$A = \int_2^x y \, dx$

$A = \int_1^3 t^4 + 2t^2 \, dx$

Given $x = 4t^3 - t^2$

$\frac{dx}{dt} = 12t^2 - 2t$

$dx = (12t^2 - 2t) \, dt$

$A = \int_1^3 t^4 + 2t^2 (12t^2 - 2t) \, dt$

$A = \int_1^3 12t^4 - 2t^5 + 24t^3 - 4t^3 \, dt$

$\left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right]$

$= \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4} \right] - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - \frac{4(1)^4}{4} \right]$

$\left[\frac{26244}{7} - 1053 \right] - \left[\frac{544}{105} \right] \left[\frac{12}{7} - \frac{2}{6} + \frac{24}{5} - 1 \right]$

$\left[\frac{18373}{7} \right] - \left[\frac{544}{105} \right]$

$= 2690.96$

3) If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, find dy/dx

Solution

$\frac{dx}{dt} = 12t^2 - 2t$ $\qquad$ $\frac{dy}{dt} = 4t^3 + 4t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$