MECHATRONICS ENGINEERING
19/ENG05/020

MAT 104

1. $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x$

$du/dx = \cos x \qquad v = e^x$

$\int u\,dv = uv - \int v\,du$

$\int u\,dv = \sin x\,(e^x) - \int e^x (\cos x)$

$\int u\,dv = e^x \sin x + \int e^x \cos x$

$\int u\,dv = e^x \sin x +$ $\quad u = \cos x \qquad dv = e^x$

$du/dx = -\sin x \qquad v = e^x$

$\int u\,dv = e^x \sin x - (e^x \cos x - \int e^x (-\sin x)\,dx)$

$\int u\,dv = e^x \sin x - e^x \cos x + \int e^x \sin x \, dx$

Let $I = \int e^x \sin x \, dx$

$I = e^x \sin x - e^x \cos x + I$

$2I = e^x \sin x - e^x \cos x$

$I = \dfrac{e^x \sin x - e^x \cos x}{2}$

$\therefore \int e^x \sin x \, dx = \dfrac{e^x \sin x - e^x \cos x + c}{2}$

b. $2x^3 \ln x$

$2x^2 \ln x \, dx$ $\quad u = \ln x \quad dv = 2x^2$

$du/dx = 1/x \quad v = \frac{2x^3}{3}$

$\int u\,dv = uv - \int v \, du/dx$

$\int u\,dv = 2x^3/3 \cdot \ln x - \int \frac{2x^3}{3} \cdot 1/x \, dx$

$\int u\,dv = 2x^3/3 \ln x - 2/3 \int \frac{x^3}{x} \, dx$

$\int u\,dv = 2x^3/3 \ln x = 2/3 \int x^2 \, dx$

$\int u\,dv = \frac{2x^3}{3} \ln x - \frac{2}{3}\left(\frac{x^3}{3}\right) + c$

$\int u\,dv = \frac{2x^3}{3} \ln x - \frac{2}{9} x^3 + c$

$\int u\,dv = \frac{2x^3}{3} \ln x - \frac{2}{9} x^3 + c$

1- $\int x^2 \sin x \, dx$

$u = x^2$ $\quad dv = \sin x$

$du/dx = 2x$ $\quad v = \int \sin x = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int u\,dv = x^2(-\cos x) - \int -\cos x\,(2x)\,dx$

$\int u\,dv = -x^2\cos x + \int 2x\cos x \, dx$

$\int u\,dv = -x^2\cos x + \left[\text{where } u = 2x \quad du/dx = 2 \right.$
$\left. dv = \cos x \quad v = \int \cos x = \sin x \right]$

$\int u\,dv = -x^2\cos x + 2x(\sin x) - \int 2(\sin x)\,dx$

$\int u\,dv = -x^2\cos x + 2x\sin x - \left[\text{where } u = 2 \quad du/dx = 0 \right.$
$\left. dv = \sin x \quad v = -\cos x \right]$

$\int u\,dv = -x^2\cos x + 2x\sin x - 2(-\cos x) - \int -\cos x\,(0)$
$+ c$

$\int x \cos x \, dx$

$u = x \qquad dv = \cos x$

$du/dx = 1 \qquad v = \int \sin x$

$$\int u \, dv = uv - \int v \, du$$

$$= x(\sin x) - \int \sin x \, dx$$

$$= x \sin x + \cos x + C$$