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Course Title: mat 104
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Assignment For Mr Okunlola
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Questions and Answers:
Find the integral of the following
1. ~~(3x-1)/(x-1)(x-2)(x-3) dx~~ $\int e^x \sin x \, dx$.

Solution

$\int e^x \sin x \, dx$ using integration by parts ie $uv - \int v du$;
According to LIATE, $u = \sin x$ and, $du = \cos x dx$, $dv = e^x dx$, $v = e^x$
$\therefore \int e^x \sin x \, dx = uv - \int v du = e^x \sin x - \int e^x \cos x \, dx$
Consider $\int e^x \cos x \, dx$, $u = \cos x$, $du = -\sin x \, dx$ $v = e^x$, $dv = e^x dx$
Substituting into the integration formula ie $uv - \int v du$.
$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$ [Note the +ve]
from $e^x \sin x - \int e^x \cos x \, dx$
$\therefore \int e^x \sin x \, dx = e^x \sin x - [e^x \cos x + \int e^x \sin x \, dx]$
$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$]
Collecting like terms
$\int e^x \sin x \, dx + \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$
$2 \int e^x \sin x \, dx = e^x \sin x + e^x \cos x$
Divide both sides by the coeffient of $\int e^x \sin x \, dx$

$$\frac{\cancel{2} \int e^x \sin x \, dx}{\cancel{2}} = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2}$$

$$\int e^x \sin x \, dx = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2} + c$$

2) $2x^2 \ln x \, dx$

Solution

$\int 2x^2 \ln x \, dx$ using Integration by parts ie $uv - \int v du$
$u = \ln x$, $dv = 2x^2 dx$ According to LIATE

$du = \frac{dx}{x}$, $v = \int 2x^2 dx = \frac{2x^3}{3}$

$\therefore$ Substituting into: $uv - \int v du$ . to get the value of $\int 2x^2$ index, we have:

$$\int 2x^2 \ln x \, dx = \frac{2}{3}x^3 \ln x - \int \frac{2}{3} x^{\cancel{3}\,2} \frac{dx}{\cancel{x}},$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3}x^3 \ln x - \int \frac{2}{3} x^2 \, dx$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^2 dx$$

$$\int 2x^2 \ln x \, dx = \frac{2}{3}x^3 \ln x - \frac{2}{3}\left[\frac{x^3}{3}\right] = \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3$$

$$\therefore \int 2x^2 \ln x \, dx = \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + C,$$

3) $x^2 \sin x \, dx$

Solution

$\int x^2 \sin x \, dx$ using integration by parts.

$\therefore u = x^2$ and $dv = \sin x \, dx$, $\frac{du}{dx} = 2x \therefore du = 2dx$

$v = -\cos x \; [\int \sin x \, dx]$

from $uv - \int v du$

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int -\cos x \, 2x \, dx \quad --- (1)$$

Arranging it by simplifying;

$$\int x^2 \sin x \, dx = -x^2 \cos x + \int 2x \cos x \, dx$$

But consider $\int 2x \cos x \, dx$, $u = 2x$ $dv = \cos x \, dx$

$\frac{du}{dx} = 2$, $du = 2dx$, $v = \sin x$

$uv - \int v du = 2x \sin x - \int 2 \sin x \, dx$

$$\int 2x \cos x \, dx = 2x \sin x - 2\int \sin x \, dx$$

$$\int 2x \cos x \, dx = 2x \sin x + 2 \cos x$$

Substituting into (1)

$\int x^2 \sin x \, dx = -x^2 \cos x - \int -\cos x \, 2x \, dx.$

$\int x^2 \sin x \, dx = -x^2 \cos x + \int 2x \cos x \, dx$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

4) $x \cos x \, dx$

<u>Solution</u>

$\int x \cos x \, dx$, integrating by parts

$u = x$, $dv = \cos x \, dx$

$\frac{du}{dx} = 1$, $du = dx$, $v = \sin x$ $[\int \cos x \, dx]$

$\therefore$ from $uv - \int v du$

$\int x \cos x \, dx = uv - \int v du = x \sin x - \int \sin x \, dx.$

$\int x \cos x \, dx = x \sin x - (-\cos x) = x \sin x + \cos x + C$