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Mechanical Engineering

$$u_0 = C \frac{du}{dx} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} [u_{i,j+1} - 2u_{i,j} + u_{i,j-1}] \Rightarrow \Delta t (\Delta x^2)$$

$$u(x, 0) = 1 \text{ at } k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial condition

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} = r u_{i+1,j} + (1-2r) u_{i,j} + r u_{i-1,j} \quad \text{--- } r^*$$

where $r = k \times \Delta t = 0$

$$u_{1,1} = 0.5 u_{1,0} + 0.5 u_{2,0}$$

$$u_{1,1} = 0.0128$$

when $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$

when $i=3, j=0$

$$= u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0})$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$u_{3,1} = 0.2176$$

$$\text{when } i=4, j=0 \Rightarrow u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0}) \\ = 0.5(0.6256) + 0.5(0.4096) \\ u_{4,1} = 0.5648$$

for $j=1$

$$\text{when } i=1: u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1}) \\ = 0.5(0) + 0.5(0.0656) \\ u_{1,2} = 0.0328$$

$$\text{when } i=2: u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1}) \\ = 0.5(0.0128) + 0.5(0.2176) \\ u_{2,2} = 0.1152$$

$$\text{when } i=3: u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1}) \\ = 0.5(0.0856) + 0.5(0.5648) \\ u_{3,2} = 0.3152$$

$$\text{when } i=4: u_{4,2} = 0.5(u_{3,1}) + 0.5(u_{5,1}) \\ = 0.5(0.2176) + 0.5(0.5648) \\ u_{4,2} = 0.6088$$

For $j=2$

$$\text{when } i=1: u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2}) \\ = 0 + 0.5(0.1152) = 0.0576$$

$$\text{when } i=2: u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2}) \\ = 0.5(0.0328) + 0.5(0.3152) = 0.174$$

$$\text{when } i=3: u_{3,3} = 0.5(u_{2,2}) + 0.5(u_{4,2}) \\ = 0.5(0.1152) + 0.5(0.6088) = 0.382$$

$$\text{when } i=4: u_{4,3} = 0.5(u_{3,2}) + 0.5(u_{5,2}) \\ = 0.5(0.3152) + 0.5(0.6088) \\ u_{4,3} = 0.6576$$

For $j=3$

$$\text{when } i=1: u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3}) \\ = 0.5(0) + 0.5(0.0576) \\ u_{1,4} = 0.0288$$

when $i=2 \Rightarrow u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{0,5})$
 $= 0.5(0.0816) + 0.5(0.362) = 0.2098$

when $i=3 \Rightarrow u_{3,4} = 0.5(u_{2,5}) + 0.5(u_{4,3})$
 $= 0.5(0.174) + 0.5(0.6576) = 0.4158$

when $i=4 \Rightarrow u_{4,4} = 0.5(u_{3,3}) + 0.5(u_{5,3})$
 $= 0.5(0.362) + 0.5(1)$
 $= 0.681$

for $J=4$
 when $j=1 \Rightarrow u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{2,4})$
 $= 0.5(2098) = 0.1049$

when $j=2 \Rightarrow u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$
 $= 0.5(0.087) + 0.5(0.4158) = 0.2514$

when $j=3 \Rightarrow u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$
 $= 0.5(0.2098) + 0.5(0.681)$
 $= 0.4454$

when $j=4 \Rightarrow u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$
 $= 0.5(0.4158) + 0.5(1)$
 $= 0.7079$

Δt	J / Temp($^{\circ}C$)						
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.651	1
0.06	3	0	0.0516	0.174	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.2056	0.2176	0.5648	1
0	0	0	0.4016	0.028	0.1296	0.4096	1
Δx		0	0.2	0.4	0.6	0.8	1
		0	1	2	3	4	5