



DAKOTA STATE  
17/ENG 06/068  
MECHANICAL ENG 382

$$7u_{i,j+1} = u_{ij} + r[u_{i+1,j} - 2u_{ij} + u_{i-1,j}] + (u_{i,j-1})$$

$$u_t = (u_{xx}) = 0$$

$$\frac{dy}{dt} - c \frac{d^2y}{dx^2} = 0$$

$$u_{ij} = 0.5(u_{i,j+1}) + 0.5(u_{i,j-1})$$

$$\frac{u_{ij+1} - u_{ij}}{\Delta t} = c \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i+1,j} = 0.5(u_{i,j+1}) + 0.5(u_{i,j-1}) + 2u_{ij} - u_{i-1,j}$$

$$u_{i+1,j} - u_{ij} = \frac{c \Delta t}{\Delta x^2} (u_{i+1,j} - 2u_{ij} + u_{i-1,j})$$

$$u_{i+1,j} = 0.0128$$

$$+ (u_{i,j-1}) \quad \Delta t / \Delta x^2 = 1$$

$$\text{when } i=2, j=0$$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6) + 0.5(0.128)$$

$$u_{2,1} = 0.864$$

$$u(x_0) = x^4/k$$

$$\Delta x = 0.2m, \Delta t = 0.01 \text{ day}$$

$$\text{when } i=3, j=0$$

$$u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0})$$

$$= 0.5(0.128) + 0.5(0.16)$$

$$u_{3,1} = 0.144$$

For Initial conditions

$$\text{At } x=0 + x^4 = 0$$

$$\text{when } i=4, j=0$$

$$u_{4,1} = 0.5648$$

$$\text{At } x=0.3 = 0.3^4 = 1.6 \times 10^{-3}$$

$$\text{For } j=1$$

$$\text{when } i=1: u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1})$$

$$= 0.5(0) + 0.5(0.864)$$

$$u_{1,2} = 0.432$$

$$\text{At } x=0.4 = 0.4^4 = 0.256$$

$$\text{when } i=2: u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$$

$$= 0.5(0.864) + 0.5(0.144)$$

$$u_{2,2} = 0.504$$

$$\text{At } x=0.6 = 0.6^4 = 0.1296$$

$$\text{when } i=3: u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1})$$

$$= 0.5(0.432) + 0.5(0.5648)$$

$$u_{3,2} = 0.4984$$

$$\text{At } x=0.8 = 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\text{when } i=3, u_{3,2} = 0.3152$$

$$\text{For } i=4$$

$$\text{when } i=1: u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2})$$

$$= 0.5(0) + 0.5(0.504)$$

$$u_{1,3} = 0.252$$

$$\text{when } i=4, u_{4,2} = 0.6088$$

$$\text{when } i=2: u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2})$$

$$= 0.5(0.432) + 0.5(0.4984)$$

$$u_{2,3} = 0.4652$$

$$\text{For } i=3$$

$$\text{when } i=1: u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$$

$$= 0.5(0) + 0.5(0.4652)$$

$$u_{1,4} = 0.2326$$

$$\text{when } i=2: u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$$

$$= 0.5(0.2326) + 0.5(0.174)$$

$$u_{2,4} = 0.2033$$

$$\text{when } i=3: u_{3,3} = 0.4454$$

$$\text{when } i=3: u_{3,3} = 0.362$$

$$\text{when } i=4: u_{4,3} = 0.7079$$

$$\text{when } i=4: u_{4,3} = 0.6526$$

For  $i=3$

$$\text{when } i=1: u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$$

$$= 0.5(0) + 0.5(0.4652)$$

$$u_{1,4} = 0.2326$$

$$\text{when } i=2: u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$$

$$= 0.5(0.2326) + 0.5(0.362)$$

$$u_{2,4} = 0.2973$$

$$\text{when } i=3: u_{3,4} = 0.4158$$

$$\text{when } i=4: u_{4,4} = 0.8914$$

| $j \setminus i$ | 0 | 1      | 2      | 3      | 4      |
|-----------------|---|--------|--------|--------|--------|
| 5               | 0 | 0.4096 | 0.2514 | 0.4158 | 0.7079 |
| 4               | 0 | 0.087  | 0.2094 | 0.4158 | 0.681  |
| 3               | 0 | 0.0536 | 0.124  | 0.362  | 0.6526 |
| 2               | 0 | 0.328  | 0.4152 | 0.3152 | 0.6088 |
| 1               | 0 | 0.428  | 0.0656 | 0.2126 | 0.5648 |
| 0               | 0 | 0.0016 | 0.028  | 0.116  | 0.4096 |
|                 | 0 | 0.2    | 0.4    | 0.6    | 0.8    |
|                 | 0 | 1      | 2      | 3      | 4      |