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 MATHS 104 GENERAL MATHEMATICS III  
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Integrate the following

1.

$$\int \frac{8x-1}{(x-1)(x-2)(x-3)} dx$$

$$\Rightarrow \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)} = \frac{8x-1}{(x-1)(x-2)(x-3)}$$

$$A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = 8x-1$$

$$A(x^2 - 5x + 6) + B(x^2 - 3x + 2) + C(x^2 + 2x + 2) = 8x - 1$$

$$A + B + C = 0 \quad \dots \textcircled{1}$$

$$-5A - 3B + 2C = 8 \quad \dots \textcircled{2}$$

$$6A + 2B + 2C = -1 \quad \dots \textcircled{3}$$

Compare  $\textcircled{2}$  with  $\textcircled{3}$

$$5A + C = 6A + 1$$

$$A = 2$$

from  $\textcircled{2}$  and  $\textcircled{3}$

$$3B + 2C = -5(2) - 8 = -18 \quad \therefore A + B + C = 0$$

$$-3C - 6 + 2C = -18$$

$$B = -C - 2$$

$$-C - 6 = -18$$

$$C = 18 - 6 = 12$$

$$A + B + C = 0$$

$$B = -A - C = -2 - 12$$

$$B = -14$$

$$\int \frac{8x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{2}{x-1} dx - \int \frac{14}{x-2} dx + \int \frac{12}{x-3} dx$$

2

$$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$$

$$\rightarrow \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{(x^2+x+1)}{(x+2)(x^2+1)}$$

$$A(x^2+1) + (Bx+C)(x+2) = x^2+x+1$$

$$A(x^2+1) + (Bx^2+2Bx+Cx+2C) = x^2+x+1$$

$$A+B = 1 \quad \text{--- (1)}$$

$$2B+C = 1 \quad \text{--- (2)}$$

$$A+2C = 1 \quad \text{--- (3)}$$

Eqn (1) into (2)

$$B = 1 - A$$

$$2(1-A) + C = 1$$

$$C = 1 - 2(1-A) = 1 + 2(A-1)$$

(2) into (3)

$$A + 2[1 + 2(A-1)] = 1$$

$$A + 2 + 4A - 4 = 1$$

$$5A - 2 = 1$$

$$5A = 3$$

$$A = \frac{3}{5}$$

$$B = 1 - A = \frac{2}{5}$$

$$C = 1 - 2B = \frac{1}{5}$$

$$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx = \int \frac{\frac{3}{5}}{x+2} dx + \int \frac{2x+1}{5(x^2+1)} dx$$

$$= \frac{3 \ln|x+2|}{5} + \frac{\ln|x^2+1|}{5} + C$$

3

$$\frac{(x^2+1)}{(x-3)(x-2)^2}$$

$$\rightarrow \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} = \frac{x^2+1}{(x-3)(x-2)^2}$$

$$x(x-2)^2 + B(x-2)(x-3) + C(x-3) = (x^2+1)(x-3)$$

$$x(x^2+4x+4) + B(x^2-5x+6) + C(x-3) = x^3+1$$

$$x + B = 1$$

$$4x + 5B - C = 0$$

$$4A + 6B - 3C = 1$$

Eqn ① into ②

$$4 - 4B + 5B - C = 0$$

$$B - C = -4 \quad \dots \text{④}$$

$$4 - 4B + 6B - 3C = 1$$

$$2B - 3C = -3 \quad \dots \text{⑤}$$

④ into ⑤

$$2C - 8 - 3C = -3$$

$$C = -5$$

$$B = C - 4 = -9$$

$$x = 1 - B = 10$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{(x-3)} + \frac{-9}{(x-2)} - \frac{5}{(x-2)^2}$$

4

$$\int \frac{(x^3 + x^2 + x + 1) dx}{(x-1)}$$

$$\Rightarrow \frac{x^2 + 2x + 3}{x-1} \int \frac{x^3 + x^2 + x + 1}{x-1}$$

$$\begin{array}{r} x^3 - 2x^2 \\ \underline{2x^2 + x + 1} \\ 2x^0 - 2x \\ \underline{5x + 1} \\ 3x - 3 \end{array}$$

$$\int \frac{x^3 + x^2 + x + 1}{x-1} dx = \int \left( x^2 + 2x + 3 + \frac{4}{x-1} \right) dx$$

$$\begin{aligned} & \int x^2 dx + \int 2x dx + \int 3 dx + \int \frac{4}{x-1} dx \\ &= \frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4 \ln|x-1| + C \end{aligned}$$