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EXAMPLES OF LINEAR TRANSFORMATIONS

Shear transformations

1

A =

"

1 0

1 1

#

A =

"

1 1

0 1

#

In general, shears are transformation in the plane with the property that there is a vector ~w such

that T(~w) = ~w and T(~x) − ~x is a multiple of ~w for all ~x. Shear transformations are invertible,

and are important in general because they are examples which cannot be diagonalized.

Scaling transformations

2

A =

"

2 0

0 2

#

A =

"

1/2 0

0 1/2

#

One can also look at transformations which scale x differently then y and where A is a diagonal

matrix. Scaling transformations can also be written as A = \_I2 where I2 is the identity matrix.

They are also called dilations.

Reflection

3

"A =

cos(2\_) sin(2\_)

sin(2\_) −cos(2\_)

#

A =

"

1 0

0 −1

#

Any reflection at a line has the form of the matrix to the left. A reflection at a line containing

a unit vector ~u is T(~x) = 2(~x · ~u)~u − ~x with matrix A =

"

2u2

1 − 1 2u1u2

2u1u2 2u2

2 − 1

#

Reflections have the property that they are their own inverse. If we combine a reflection with

a dilation, we get a reflection-dilation.

Projection

4

A =

"

1 0

0 0

#

A =

"

0 0

0 1

#

A" projection onto a line containing unit vector ~u is T(~x) = (~x · ~u)~u with matrix

#

.

Projections are also important in statistics. Projections are not invertible except if we project

onto the entire space. Projections also have the property that P2 = P. If we do it twice, it

is the same transformation. If we combine a projection with a dilation, we get a rotation

dilation.

Rotation

5

A =

"

−1 0

0 −1

# "A =

cos(\_) −sin(\_)

sin(\_) cos(\_)

#

Any rotation has the form of the matrix to the right.

Rotations are examples of orthogonal transformations. If we combine a rotation with a dilation,

we get a rotation-dilation.

Rotation-Dilation

6

A =

"

2 −3

3 2

#

A =

"

a −b

b a

#

A rotation dilation is a composition of a rotation by angle arctan(y/x) and a dilation by a

factor px2 + y2.

If z = x + iy and w = a + ib and T(x, y) = (X, Y ), then X + iY = zw. So a rotation dilation

is tied to the process of the multiplication with a complex number.

Rotations in space

7

Rotations in space are determined by an axis of rotation and an

angle. A rotation by 120\_ around a line containing (0, 0, 0) and

(1, 1, 1) belongs to A =

2

64

0 0 1

1 0 0

0 1 0

3

75

which permutes ~e1 !~e2 !~e3.

Reflection at xy-plane

8

To a reflection at the xy-plane 2 belongs the matrix A =

64

1 0 0

0 1 0

0 0 −1

3

75

Projection into space

9

To project a 4d-object into the three dimensional xyz-space, use

for example the matrix A =

2

6664

1 0 0 0

0 1 0 0

0 0 1 0

0 0 0 0

3

7775