$\lim_{x \to 0} \frac{(x - \cos x)}{x}$

By L'Hopital's rule, we have

$= \lim_{x \to 0} \frac{(1 - (-\sin x))}{1}$

$= \lim_{x \to 0} \frac{(1 + \sin x)}{1} = \frac{(1 + \sin(0))}{1} = \frac{1+0}{1} = \frac{1}{1} = 1$

2. If $y = -3\tan 7x e^{3x}$, find $dy/dx$

Solution

$u = -3\tan 7x \qquad v = e^{3x}$

$\frac{du}{dx} = -21\sec^2 7x \qquad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = (-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)$

$\frac{dy}{dx} = [(-3\tan 7x)(3e^{3x}) + e^{3x}(-21\sec^2 7x)]$

$\frac{dy}{dx} = [3e^{3x}(-3\tan 7x) + e^{3x}(-21\sec^2 7x)]$

3- If $y = \cos 3x$, find $dy/dx$ from the first principle

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos[3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

Collecting the like terms

$$= \left[\left(\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right)\right] - \left[\left(\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos(3\Delta x) - 1}{\Delta x} \times \frac{\cos(3\Delta x + 1)}{\cos(3\Delta x + 1)}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\cos 3x\left(\frac{\cos^2(3\Delta x) - 1}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

From the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$$

$$= \lim_{\Delta x \to 0}\left[-\cos 3x\left(\frac{\sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x}\right)\right] - \left[\sin 3x\left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x) - \sin 3x}{\cos(3\Delta x + 1)\Delta x}\right]\left[\frac{\sin(3\Delta x)}{\Delta x}\right]$$

6. Find the gradient of $x^2 + 2xy + y^2 = 1020$

Solution

$$x^2 + 2xy + y^2 = 1020$$

$$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$$

$$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$$

$$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx}(2x + 2y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$$

$\therefore$

7. First darvatue of the fumetun $y = x^2 \cos x$

Solutun

$$y = x^2 \cos x$$

$$u = x^2, \quad v = \cos x$$

$$\frac{du}{dx} = 2x, \quad \frac{dv}{dx} = -\sin x$$

Using product rule

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2)(-\sin x) + (2x)(\cos x)$$

$$= -x^2 \sin x + 2x\cos x$$

$$= [2x\cos x - x^2 \sin x]$$

$\therefore$

$$= \left(\lim_{\Delta x \to 0}\left[\frac{-\cos 3x \sin^2(3\Delta x)}{\cos(3\Delta x+1)\Delta x} - \sin 3x\right]\right) = \left(\lim_{\Delta x \to 0}\left[\frac{\sin(3\Delta x)}{\Delta x}\right]\right)$$

First limit → $-\left(\frac{\cos 3x \sin^2(3(0))}{\cos(0+1)0}\right) - \sin 3x$

Rewritten as → $-\sin 3x \lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x}$

Recall $\lim_{\Delta x \to 0} \frac{\sin x}{x} = 1$, therefore $\lim_{\Delta x \to 0} \frac{\sin(3\Delta x)}{\Delta x} = 3$

Final form ends up as $(-\sin 3x)(3)$

$= -3\sin 3x$

4.) $(f-g)(5) = f(5) - g(5)$

$(f-g)(5) = [2(5)^3 - 7(5)] - [-3(5)]$

$(f-g)(5) = [2(125) - 35] - [-15]$

$(f-g)(5) = [250 - 35] + 15$

$(f-g)(5) = 215 + 15$

$(f-g)(5) = 230$

5.) Find $fog(x)$ if

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

Solution

$fog(x) = 4(2x+3)^2 + 2$

$= 4(2x+3)(2x+3) + 2$

$= 4(4x^2 + 6x + 6x + 9) + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$