

Assignment

① $x^2 \sin x \, dx$

solution

$$\int x^2 \sin x \, dx$$

$$u = x^2 \quad dv = \sin x$$

$$du = 2x \, dx \quad v = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$= x^2(-\cos x) - \int (-\cos x)(2x) \, dx$$

$$= -x^2 \cos x - \int -2x \cos x \, dx$$

$$\int -2x \cos x \, dx$$

$$u = -2x \quad dv = \cos x$$

$$du = -2 \, dx \quad v = \sin x$$

$$\int u \, dv = uv - \int v \, du$$

$$= (-2x)(\sin x) - \int (\sin x)(-2) \, dx$$

$$= -2x \sin x - \int -2 \sin x \, dx$$

$$= -2x \sin x - (-2) \int \sin x \, dx$$

$$= -2x \sin x + 2 \int \sin x \, dx$$

$$= -2x \sin x + 2(-\cos x)$$

$$= -2x \sin x - 2 \cos x$$

$$\therefore \int x^2 \sin x \, dx = -x^2 \cos x - [-2x \sin x - 2 \cos x]$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

4

$$\begin{aligned}
&= -3 \ln(1-x) + 6(1-x) - \frac{3(1-x)^2}{2} \\
&= -3 \ln(1-x) + \frac{6(1-x)}{1} - \frac{3(1-2x+x^2)}{2} \\
&= -3 \ln(1-x) + \frac{(6 \times 2)(1-x) - 3(1-2x+x^2)}{2} \\
&= -3 \ln(1-x) + \frac{12(1-x) - (3-6x+3x^2)}{2} \\
&= -3 \ln(1-x) + \frac{12-12x-3+6x-3x^2}{2}
\end{aligned}$$

$$\int \frac{3x^2}{1-x} dx = -3 \ln(1-x) + \frac{9-6x-3x^2}{2}$$

So,

$$\begin{aligned}
\int \frac{2x}{1-x} dx - \int \frac{3x^2}{1-x} dx &= -2 \ln(1-x) + 2-2x - \left[-3 \ln(1-x) + \frac{9-6x-3x^2}{2} \right] \\
&= -2 \ln(1-x) + 2-2x + 3 \ln(1-x) - \frac{9+6x+3x^2}{2} \\
&= \frac{2-2x}{1} - \frac{9+6x+3x^2}{2} - 2 \ln(1-x) + 3 \ln(1-x) \\
&= \frac{2(2-2x) - 9+6x+3x^2}{2} + \ln(1-x) + C \\
&= \frac{4-4x-9+6x+3x^2}{2} + \ln(1-x) + C
\end{aligned}$$

$$\int \frac{2x-3x^2}{(1-x)} dx = \frac{3x^2+2x-5}{2} + \ln(1-x) + C$$

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