Okiemute Waterway
17/ENG03/056
Civil Engineering

$$u_t - Cu_{xx} = 0$$

$$\frac{\partial u}{\partial t} - C\frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C\cdot\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C\frac{\Delta t}{\Delta x^2}\left[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right]$$

$$\Delta t/\Delta x^2, \quad r = 1$$

$$u[x,0] = x^4/k$$

$\Delta x = 0.2\,m$, $\Delta t = 0.2\,day$

for initial conditions

At $x = 0 \rightarrow x^4 = 0$

At $x = 0.2 \rightarrow 0.2^4 = 1.6\times10^{-3}$

At $x = 0.4 \rightarrow 0.4^4 = 0.0256$

At $x = 0.6 \rightarrow 0.6^4 = 0.1296$

At $x = 0.8 \rightarrow 0.8^4 = 0.4096$

At $x = 1 \longrightarrow 1^4 = 1$

$$\therefore u_{i,j+1} = u_{i,j} + r\left[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right]$$

$$u_{i,j+1} = r\,u_{i-1,j} + (1-2r)u_{i,j} + r\,u_{i+1,j} \quad \text{---} *$$

when $i = 1$, $j = 0$

$$u_{1,1} = 0.5u_{0,0} + 0.5u_{2,0}$$

$= 0.5(0) + 0.5(0.0256)$

$u_{1,1} = 0.0128$

when $i = 2, j = 0$

$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$

$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$

$u_{2,1} = 0.0656$

when $i = 3, j = 0$

$u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0})$

$= 0.5(0.0256) + 0.5(0.4096)$

$u_{3,1} = 0.2176$

when $i = 4, j = 0$

$u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0})$

$= 0.5(0.0256) + 0.5(0.4096)$

$u_{4,1} = 0.5648$

For $j = 1$

when $i = 1$ $\therefore u_{1,2} = 0.5(0_{1,1}) + 0.5(u_{2,1})$

$= 0.5\, u_{0,1} + 0.5\, u_{2,1} = 0 + 0.5(0.0656)$

$u_{1,2} = 0.0328$

when $i = 2$ $\therefore u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$

$= 0.5(0.0128) + 0.5(0.2176)$

$u_{2,2} = 0.1152$

when $i = 3$ $u_{3,2} = 0.5(u_{0,1}) + 0.5(u_{4,1})$

$0.5(0.0656) + 0.5(0.5649)$

$u_{3,2} = 0.3152$

when $i = 4$ $\therefore u_{4,2} = 0.5(u_{2,1}) + 0.5(u_{5,1})$

$= 0.5(0.2176) + 0.5(1)$

$u_{4,2} = 0.6088.$

For $j = 2$

when $i = 1$

$$u_{1,3} = 0.5(u_{0,1}) + 0.5(u_{2,2})$$
$$= 0 + 0.5(0.1152) = 0.0576$$

when $i = 2$ $\quad u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2})$
$$= 0 + 0.5(0.0328) + 0.5(0.3152)$$
$$= 0.174$$

when $i = 3$

$$u_{3,3} = 0.5(u_{2,2}) + 0.5(u_{4,2})$$
$$= 0.5(0.1152) + 0.5(0.6088)$$
$$= 0.362$$

when $i = 4$ $\quad = u_{4,4} = 0.5(u_{3,2}) + 0.5(u_{5,2})$
$$= 0.5(0.3152) + 0.5(1)$$
$$u_{4,3} = 0.6576.$$

For $j = 3$

when $i = 1$

$$u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$$
$$= 0.5(0) + 0.5(0.174) = 0.084$$

when $i = 2$ $\quad u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$
$$= 0.5(0.0576) + 0.5(0.362)$$
$$= 0.2098$$

when $i = 3$ $\quad u_{3,4} = 0.5(u_{2,3}) + 0.5(u_{4,3})$
$$= 0.5(0.174) + 0.5(0.6576) =$$
$$= 0.4158$$

when $i = 4$ $\quad u_{4,4} = 0.5(u_{3,3}) + 0.5(u_{5,3})$
$$= 0.5(0.362) + 0.5(1) = 0.681$$

For $j = 4$

when $i = 1$ $\quad u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{2,4})$

$= 0.5(0.2098) = 0.1049$

when $i = 2$ $\quad u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$

$= 0.5(0.087) + 0.5(0.4158)$

$= 0.2514$

when $i = 3$ $\quad u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$

$0.5(0.2098) + 0.5(0.681)$

$= 0.4454$

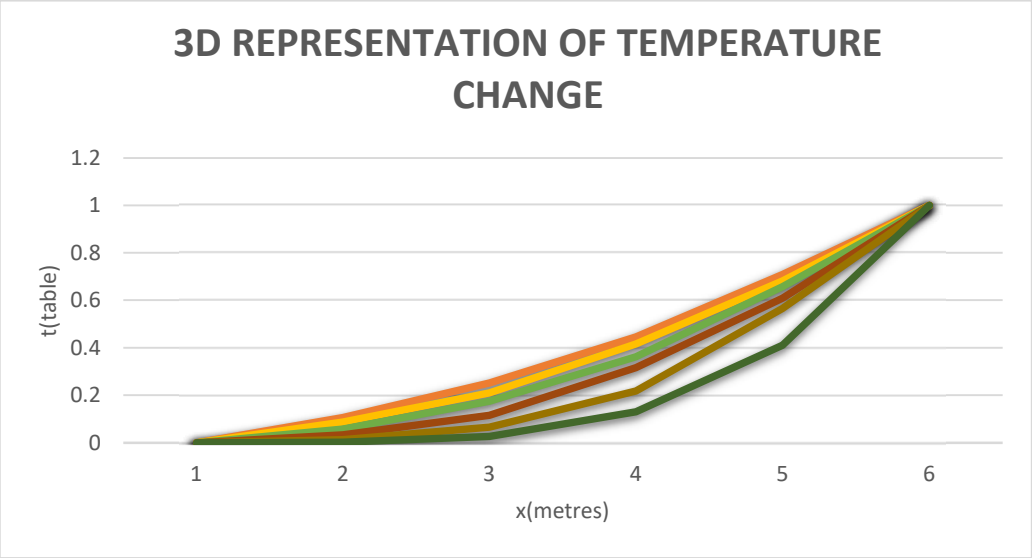
when $i = 4$ $\quad u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$

$0.5(0.4158) + 0.5(1)$

$= 0.7079$

Δt \ J / Temp(x)							
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.03	3	0	0.0576	0.174	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0	0.0016	0.0128	0.1296	0.4096	1
Δx		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1



0.5