$$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$$

$$\therefore \Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-12x - 6(\Delta x)}{x^4}$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right]$$

$$\therefore \frac{du}{dx} = \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$$

$$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$$

Putting the value of u back

$$\therefore \frac{dy}{dx} = \frac{-12\cos(6/x^2)}{x^3}$$

MAT 104 Assignment

Answers

1.) $y = \sin\left(\frac{6}{x^2}\right)$

Let $u = 6/x^2$ ----- (1)

$\therefore y = \sin u$

$y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u/2}$

$\therefore \frac{\Delta y}{\Delta u} = \frac{\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)}{\Delta u/2}$

$\lim_{\Delta u \to 0}\left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \to 0}\left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \cdot \lim_{\Delta u \to 0}\left[\frac{\sin(\Delta u/2)}{\Delta u/2}\right]$

$\therefore \frac{dy}{du} = \cos u$

From Equation (1), $u = 6/x^2$

$u + \Delta u = \frac{6}{(x + \Delta x)^2}$

$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$