ASITA OBONISO
17/ENG06/014 MECHATRONICS ENGINEERING

ENGINEERING MATHS ENG 382

ASSIGNMENT U1

$$u_t - Cu_{xx} = 0$$

$$\frac{du}{dt} - C\frac{d^2u}{dx^2} = 0$$

Making $\frac{du}{dt}$ the subject

$$\frac{du}{dt} = C\frac{d^2u}{dx}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{C(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{C\Delta t}{\Delta x^2}(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) \quad r = \frac{\Delta t}{\Delta x^2}$$

$C = 1$

$u(x,0) = x^4$

$\Delta x = 0.2m, \quad \Delta t = 0.02$ day

For initial Conditions

$x = 0 \Rightarrow x^4 = 0$

$x = 0.2 \Rightarrow 0.2^4 = 1.6 \times 10^{-3}$

$x = 0.4 \Rightarrow 0.4^4 = 0.0256$

$x = 0.6 \Rightarrow 0.6^4 = 0.1296$

$x = 0.8 \Rightarrow 0.8^4 = 0.4096$

$x = 1.0 \Rightarrow 1.0^4 = 1$

Therefore, $u_{i,j+1} = u_{i,j} + r(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$

$u_{i,j+1} = ru_{i-1,j} + (1-2r)u_{i,j} + ru_{i+1,j}$ — eqn*

When $i = 1, j = 0$

$u_{1,1} = 0.5(u_{0,0}) + 0.5(u_{2,0})$

$u_{1,1} = 0.5(0) + 0.5(0.0256)$

$u_{1,1} = 0.0128$

When $i = 2, j = 0$

$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$

$u_{2,1} = 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$

$u_{2,1} = 0.0656$

When $i = 3, j = 0$

$u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0})$

$u_{3,1} = 0.5(0.0256) + 0.5(0.4096)$

$u_{3,1} = 0.2176$

When $i = 4, j = 0$

$u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0})$

$u_{4,1} = 0.5(0.0256) + 0.5(0.4096)$

$u_{4,1} = 0.5648$

For $j = 1$

When $i = 1 \longrightarrow u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1})$

$u_{1,2} = 0.5\,u_{0,1} + 0.5\,u_{2,1} = 0 + 0.5(0.0656)$

$u_{1,2} = 0.0328$

When $i = 2 \longrightarrow u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$

$u_{2,2} = 0.5(0.0128) + 0.5(0.2176)$

$u_{2,2} = 0.1152$

When $i = 3 \longrightarrow u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1})$

$u_{3,2} = 0.5(0.0656) + 0.5(0.5648)$

$u_{3,2} = 0.3152$

When $i = 4 \longrightarrow u_{4,2} = 0.5(u_{3,1}) + 0.5(u_{5,1})$

$u_{4,2} = 0.5(0.2176) + 0.5(1)$

$u_{4,2} = 0.6088$

For $j = 2$

When $i = 1 \longrightarrow u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2})$

$u_{1,3} = 0 + 0.5(0.1152$

$u_{1,3} = 0.0576$

When $i = 2 \longrightarrow u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2})$

$u_{2,3} = 0.5(0.0328) + 0.5(0.3152)$

$u_{2,3} = 0.174$

When $i = 3 \longrightarrow u_{3,3} = 0.5(u_{2,2}) + 0.5(u_{4,2})$

$u_{3,3} = 0.5(0.1152) + 0.5(0.6088)$

$u_{3,3} = 0.362$

When $i = 4 \longrightarrow u_{4,3} = 0.5(u_{3,2}) + 0.5(u_{5,2})$

$u_{4,3} = 0.5(0.3152) + 0.5(1)$

$u_{4,3} = 0.6576$

For $j = 3$

When $i = 1 \longrightarrow u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$

$u_{1,4} = 0.5(0) + 0.5(0.174)$

$u_{1,4} = 0.084$

When $i = 2 \longrightarrow$ $u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$

$u_{2,4} = 0.5(0.0576) + 0.5(0.362)$

$u_{2,4} = 0.2098$

When $i = 3 \longrightarrow$ $u_{3,4} = 0.5(u_{2,3}) + 0.5(u_{4,3})$

$u_{3,4} = 0.5(0.174) + 0.5(0.6576)$

$u_{3,4} = 0.4158$

When $i = 4 \longrightarrow$ $u_{4,4} = 0.5(u_{3,3}) + 0.5(u_{5,3})$

$u_{4,4} = 0.5(0.362) + 0.5(1)$

$u_{4,4} = 0.681$

For $j = 4$

When $i = 1 \longrightarrow$ $u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{2,4})$

$u_{1,5} = 0.5(0.2098)$

$u_{1,5} = 0.1049$

When $i = 2 \longrightarrow$ $u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$

$u_{2,5} = 0.5(0.087) + 0.5(0.4158)$

$u_{2,5} = 0.2514$

When $i = 3 \longrightarrow$ $u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$

$u_{3,5} = 0.5(0.2098) + 0.5(0.681)$

$u_{3,5} = 0.4454$

When $i = 4 \longrightarrow$ $u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$

$u_{4,5} = 0.5(0.4158) + 0.5(1)$

$u_{4,5} = 0.7079$

Δt	j / Temp(x)						
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1
0.08	4	0	0.0870	0.2098	0.4158	0.6810	1
0.06	3	0	0.0576	0.1740	0.362	0.6576	1
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1
0.02	1	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0	0.0016	0.028	0.1296	0.4096	1
Δx		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5