

$$1. \quad y = \sin\left(\frac{b}{x^2}\right)$$

$$\text{Let } u = \frac{b}{x^2} \dots \dots \dots (1)$$

$$\therefore y = \sin u$$

$$y + \Delta y = \sin(u + \Delta u)$$

$$\Delta y = \sin(u + \Delta u) - y$$

$$\Delta y = \sin(u + \Delta u) - \sin u$$

$$\Delta y = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\frac{\Delta y}{\Delta u} = 2 \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}$$

$$\therefore \frac{\Delta y}{\Delta u} = \cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$$

$$\lim_{\Delta u \rightarrow 0} \left(\frac{\Delta y}{\Delta u}\right) = \lim_{\Delta u \rightarrow 0} \left[\cos\left(\frac{2u + \Delta u}{2}\right)\right] \cdot \lim_{\Delta u \rightarrow 0} \left[\frac{\sin\left(\frac{\Delta u}{2}\right)}{\frac{\Delta u}{2}}\right]$$

$$\therefore \frac{dy}{dx} = \cos u$$

$$\text{From Equation (1) } u = \frac{b}{x^2}$$

$$u + \Delta u = \frac{b}{(x + \Delta x)^2}$$

$$u + \Delta u = \frac{b}{x^2 + 2x(\Delta x) + (\Delta x)^2}$$

$$2. \quad x = 4t^3 - t^2, \quad y = t^4 + 2t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad dx = (12t^2 - 2t)dt$$

Let A represent the area $\int_a^b y dx$

$$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t) dt$$

$$\triangle \cdot A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3) dt$$

$$\Rightarrow \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 + C \right]_1^3$$

$$\left(\frac{26244}{7} - 243 + \frac{5832}{5} - 81 \right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1 \right)$$

$$= \frac{160704}{35} - \frac{544}{105}$$

$$\triangle \cdot A = 4586.36 \text{ sq units}$$

3. $x = 4t^3 - t^2, y = t^4 + 2t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 - 4t$$

$$\frac{dx}{dt} = 12t^2 - 2t, \quad \frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$$

$$\triangle \cdot \frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$$