$$= i\left(\frac{u^{3+1}}{4} + \frac{u^{2+1}}{3} + \frac{6u^{2}}{2}\right)\Bigg|_0^1 - j\left(\frac{u^3}{3} - \frac{10u^2}{2}\right)\Bigg|_0^1 + k\left[\frac{-9u^3}{3} - \frac{2u^4}{4}\right]_0^1$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right]_0^1 - j\left[\frac{u^3}{3} - 5u^2\right]_0^1 + k\left[-3u^3 - \frac{u^4}{2}\right]_0^1$$

$$= i\left[\left(\frac{1^4}{4} + \frac{1^3}{3} + 3\right) - \left(\frac{0^4}{4} + \frac{0^3}{3} + 0\right)\right] = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] = \frac{43}{12}i$$

$$= j\left(\frac{1^3}{3} - 5\right) - \left(\frac{0^3}{3} - 0\right) = j\left(\frac{1}{3} - 5\right) = -\frac{14}{3}j$$

$$= k\left[\left(-3(1)^3 - \frac{1^4}{2}\right) - (0 - 0)\right] = k\left(-3 - \frac{1}{2}\right) = -\frac{7}{2}k$$

$$\int_0^1 (f \times v)\,du = \frac{43i}{12} - \frac{14}{3}j - \frac{7}{2}k$$

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1.) If $\bar{M} = Pi - 6j - 3k$, $\bar{N} = 4i + 3j - k$, $\bar{O} = i - 3j + 2k$, find the value of P for which

a) $\bar{M}$ and $\bar{N}$ are perpendicular to each other.

Solution

Given $\bar{M} = Pi - 6j - 3k$ and $\bar{N} = 4i + 3j - k$

For perpendicular vectors

$\bar{M} \cdot \bar{N} = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4p - 18 + 3$

$\bar{M} \cdot \bar{N} = 4p - 15$

$\bar{M} \cdot \bar{N} = 0$

$4p - 15 = 0$

$\frac{4p}{4} = \frac{15}{4}$

$p = 15/4$

b) $\bar{M}$, $\bar{N}$ and $\bar{O}$ are coplanar.

Solution

$$\bar{M} \times (\bar{N} \times \bar{O}) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= P(6-3) - 6(8-(-1)) - 3(-12-3) = 0$

$= P(3) - 6(9) - 3(-15)$

$= 3p - 54 + 45$

$\bar{M} \times (\bar{N} \times \bar{O}) = 3p - 54 + 45 = 0$

$= 3p - 9 = 0$

$\frac{3p}{3} = \frac{9}{3}$

$p = 3$

$\therefore \bar{M} \cdot (\bar{N} \times \bar{O}) = 3(3) - 54 + 45$

$= 9 - 9$

$= 0$

$\bar{M}$, $\bar{N}$ and $\bar{O}$ are coplanar

2) Find the direction cosines and the unit vectors along the sum of $3i+2j+5k$, $2i-j+6k$ and $5i+2j-8k$

<u>Solution</u>

Let $A = 3i+2j+5k$, $\bar{B} = 2i-j+6k$ and $\bar{C} = 5i+2j-3k$

$\bar{r}$ be the sum of A, B and C

$\bar{r} = (3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$\bar{r} = 10i+3j+8k$

a) The direction cosine is given as $\cos A, \cos B, \cos C$

$|r| = \sqrt{10^2+3^2+8^2} = \sqrt{173} = 13.15$

$\cos A = \frac{}{|r|} = \frac{10}{13.15} = 0.7605$

$\cos B = \frac{}{|r|} = \frac{3}{13.15} = 0.2281$

$\cos C = \frac{}{|r|} = \frac{8}{13.15} = 0.6084$

The unit vector of $|r|$

$e_r = \frac{\bar{r}}{|r|} = \frac{10i+3j+8k}{13.15}$ or $\frac{10}{13.15}i + \frac{3}{13.15}j + \frac{8}{13.15}k$

3) If $f = 3ui + u^2j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$, evaluate the integral of $(f \times v)du$ from 0 to 1

Solution

$f = 3ui + u^2j + (u+2)k$

$V = 2ui - 3uj + (u-2)k$

$\bar{f} \times \bar{v} = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$

$\bar{f} \times \bar{v} = \cancel{i\{u^3 - 2u^2}$

$\bar{f} \times \bar{v} = i\{(u^3-2u^2)-(-3u^2+6u)\} - j\{(3u^2-6u)-(2u^2+4u)\} + k\{(-9u^2-2u^3)\}$

$= i\{u^3-2u^2+3u^2+6u\} - j\{3u^2-6u-2u^2-4u\} + k\{-9u^2-2u^3\}$

$= i\{u^3+u^2+6u\} - j\{u^2-10u\} + k\{-9u^2-2u^3\}$

$\bar{f} \times \bar{v} = (u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k$

Given $(f \times v) = (u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k$

$\int_0^1 (f \times v)du = \int_0^1 (u^3+u^2+6u)i - (u^2-10u)j + (-9u^2-2u^3)k$

Through Integrating