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MAT 204

let matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 5 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 1 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 5 & 0 \\ 3 & 7 & 1 \\ 2 & 1 & 4 \end{bmatrix}$

1) Linear transformation of A if vector $X = (a, b, c)$

$T(x) = A(x)$

$= \begin{bmatrix} 1 & 0 & 3 \\ 4 & 5 & -1 \\ 0 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$= a \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

$T(x) = \begin{bmatrix} a \\ 4a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 5b \\ 0 \end{bmatrix} + \begin{bmatrix} 3c \\ -c \\ 2c \end{bmatrix}$

$T(x) = \begin{bmatrix} a + 0 + 3c \\ 4a + 5b + (-c) \\ 0 + 0 + 2c \end{bmatrix}$

Hence, the transformation of

$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ gives $\begin{bmatrix} a + 3c \\ 4a + 5b - c \\ 2c \end{bmatrix}$

For C,

$$|C| = \begin{vmatrix} 0 & 5 & 0 \\ 3 & 7 & 1 \\ 2 & 1 & 4 \end{vmatrix}$$

$$= 0\begin{vmatrix} 7 & 1 \\ 1 & 4 \end{vmatrix} - 5\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} + 0\begin{vmatrix} 3 & 7 \\ 2 & 1 \end{vmatrix}$$

$$= 0 - 5(12 - 2) + 0$$

$$|C| = -50 \neq 0$$

$\therefore$ C is a non-singular matrix

3) For A,

$$|A| = \begin{vmatrix} 1 & 0 & 3 \\ 4 & 5 & -1 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 5 & -1 \\ 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 4 & -1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 0 & 0 \end{vmatrix}$$

$$= 1(10-0) - 0 + 0$$

$$|A| = 10 \neq 0$$

$\therefore$ A is a non-singular matrix.

For B,

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 4 & 1 & 5 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ 4 & 5 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 1(10-1) - 2(0-4) + 3(0-8)$$

$$= 9 + 8 - 24$$

$$|B| = -7 \neq 0$$

$\therefore$ B is a non-singular matrix

2) Rank of $(B+C)^T$.

$$B+C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 4 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 0 \\ 3 & 7 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$

$$B+C = \begin{bmatrix} 1 & 7 & 3 \\ 3 & 9 & 2 \\ 6 & 2 & 9 \end{bmatrix}$$

$$(B+C)^T = \begin{bmatrix} 1 & 3 & 6 \\ 7 & 9 & 2 \\ 3 & 2 & 9 \end{bmatrix}$$

Rank of $(B+C)^T$

$$|(B+C)^T| = 1\begin{vmatrix} 9 & 2 \\ 2 & 9 \end{vmatrix} - 3\begin{vmatrix} 7 & 2 \\ 3 & 9 \end{vmatrix} + 6\begin{vmatrix} 7 & 9 \\ 3 & 2 \end{vmatrix}$$

$$= 1(81-4) - 3(63-6) + 6(14-27)$$

$$= 77 - 171 - 78$$

$$|(B+C)^T| = -172 \neq 0$$

$\therefore$ The Rank is 3