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COURSE CODE: MAT 104

1) Find the Limit of the function $(4x^2 - \sin x)/x^3$ as $x \to 0$

Solution

Given $\frac{4x^2 - \sin x}{x^3}$, by direct substitution as x tends to 0, we have:

$$\frac{4(0) - \sin 0}{0^3} = \frac{0 - 0}{0} = \frac{0}{0} = \text{undefined}$$

using L'hopital's rule

$$\lim_{x \to 0}\left(\frac{4x^2 - \sin x}{x^3}\right) = \frac{8x - \cos x}{3x^2} = \frac{8x - \cos x}{3x^2}$$

i.e by differentiating

$$\therefore \lim_{x \to 0}\left(\frac{8(0) - \cos 0}{3(0)^2}\right) = \frac{0 - 1}{0} = \frac{-1}{0} = \text{undefined}$$

$\therefore$ by further differentiating

$$\frac{8x - \cos x}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}$$

$$\therefore \lim_{x \to 0}\left(\frac{8 + \sin x}{6x}\right) = \frac{8 + \sin 0}{6(0)} = \text{undefined}$$

By further differentiation

$$\frac{8 + \sin x}{6x} = \frac{\cos x}{6}$$

$$\lim_{x \to 0} = \left(\frac{\cos 0}{6}\right) = \frac{1}{6}$$

2). If $y = (7x^2 \cos 8x)/e^{3x}$, find the derivative of y with respect to x

Solution

Given $y = \frac{7x^2 \cos 8x}{e^{3x}}$

Let $u = 7x^2$, $\frac{du}{dx} = 14x$

$v = \cos 8x$, $\frac{dv}{dx} = -8\sin 8x$

$w = e^{3x}$ $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left(\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} - \frac{1}{w} \times \frac{dw}{dx}\right)$$

$$\frac{dy}{dx} = y\left(\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times -8\sin 8x - \frac{1}{e^{3x}} \times 3e^{3x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - 3\right)$$

$$\frac{dy}{dx} = y\left(2/x - 8\tan 8x - 3\right)$$

3) If $y = \cos(5x^2 + 6x)$, find dy/dx

Given $y = \cos(5x^2 + 6x)$

Let $u = 5x^2 + 6x$

$\therefore \frac{du}{dx} = 10x + 6$

$\therefore y = \cos u$

$\frac{dy}{du} = -\sin u$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 10x + 6 \times -\sin u$$

$$\frac{dy}{dx} = -\sin u \times 10x + 6$$

$$\frac{dy}{dx} = -\sin u\,(10x + 6), \text{ but } u = 5x^2 + 6x$$

$$\therefore \frac{dy}{dx} = -\sin 5x^2 + 6x\,(10x + 6)$$

4) Find the Integral of the following.

a) $\frac{3}{(4x+1)}\,dx$

Solution

$\int \frac{3}{4x+1}\,dx$, Let $u = 4x + 1$, $\frac{du}{dx} = \frac{4}{1}$

$dx = \frac{du}{4}$. but $x = \frac{u-1}{4}$

$$\therefore \int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3\,du}{4(4x+1)}$$

Integrating,

$$= \frac{3}{4}\int \frac{du}{4x+1} = \frac{3}{4}\int \frac{du}{u} = \frac{3}{4} \times \frac{u^{-1}}{4} + C$$

by final simplification, $u = \frac{3}{2u^2} + C$

$$= \frac{3}{2(4x+1)^2} + C$$

b) $\frac{dx}{(x^2+49)}$

Solution

$$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{x^2+7^2}$$

$$x = 7\tan\theta$$

$$\frac{dx}{d\theta} = 7\sec^2\theta, \quad dx = 7\sec^2\theta\, d\theta$$

$$x^2+7^2 = 7\tan^2\theta + 7^2 = 7^2(\tan^2\theta + 1)$$

Factorizing after substituting

$$\int \frac{\cancel{7\sec^2\theta}\, d\theta}{\cancel{49}\, 7 \cancel{\sec^2\theta}} = \int \frac{d\theta}{7} = \frac{1}{7}\int d\theta = \frac{1}{7}\theta + C$$

but $\theta = \tan^{-1}\frac{x}{7}$

$$= \frac{1}{7}\tan^{-1}\frac{x}{7} + C \,//$$

c) $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)dx$

Solution

$$\int e^{6x} + 9x - \sin 7x + \cos 8x\, dx$$

$$= \frac{1}{6}e^{6x} + \frac{9x^2}{2} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$$

d) $x\sqrt{9+x^2\,dx}$

Solution

$$\int x\sqrt{9+x^2}\, dx$$

let $u = 9+x^2$; $x = \sqrt{u-9}$

$$\frac{du}{dx} = 2x, \quad dx = \frac{du}{2x}$$

From $\int x\sqrt{9+x^2}\,dx$, substituting

$$\int x\, u^{1/2} \frac{du}{2x}$$

$$\int (\sqrt{u-9})\, u^{1/2} \frac{du}{2x} = \int \frac{(u-9)^{1/2} u^{1/2}\, du}{2(u-9)^{1/2}}$$

$$= \frac{1}{2}\int u^{1/2}\, du$$

Integrating

$$= \frac{1}{2}\left(\frac{u^{1/2+1}}{\frac{1}{2}+1}\right) = \frac{1}{2}\frac{u^{3/2}}{3/2}$$

$$= \frac{1}{2}\left(\frac{u^{3/2}}{1} \times \frac{2}{3}\right) = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} u^{3/2} = \frac{u^{3/2}}{3} = \frac{1}{3}u^{3/2} \,//$$