NAME: AIARI ELIJAH ENIOLA
DEPARTMENT: AERONAUTICAL & ASTRONAUTICAL ENGINEERING
COURSE CODE: MAT 204
MATRIC NO: 19/ENG05/002

1. $e^x \sin x \, dx$

Solution

$u = \sin x, \ dv = e^x dx$

$\frac{du}{dx} = \cos x, \ v = e^x$

$\int u dv = uv - \int v du$

$\int e^x \sin x \, dx = (\sin x)(e^x) - \int e^x \cos x \, dx \quad -(i)$

$\int e^x \cos x \, dx.$

$u = \cos x, \ dv = e^x dx$

$\frac{du}{dx} = -\sin x, \ v = e^x$

$\int u dv = uv - \int v du$

$\int e^x \cos x \, dx = (\cos x)(e^x) - \int e^x(-\sin x) dx$

$= e^x \cos x + \int e^x \sin x \, dx \quad -(ii)$

Put equation (ii) into (i)

$\int e^x \sin x \, dx = e^x \sin x - \boxed{e^x \cos x + \int e^x \sin x \, dx}$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

collecting like terms

$\int e^x \sin x \, dx + \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

Divide both sides by the coefficient of $\int e^x \sin x \, dx$

$\frac{\cancel{2}\int e^x \sin x \, dx}{\cancel{2}} = \frac{e^x \sin x - e^x \cos x}{2}$

$\int e^x \sin x \, dx = \frac{e^x \sin x}{2} - \frac{e^x \cos x}{2} + C$

2. $2x^2 \ln x \, dx$

Solution

$u = \ln x, \ dv = 2x^2 dx$

$\frac{du}{dx} = \frac{1}{x}, \ v = \frac{2x^3}{3}$

$\int u dv = uv - \int v du$

$\int 2x^2 \ln x \, dx = (\ln x)\left(\frac{2x^3}{3}\right) - \int\left(\frac{2x^{\cancel{3}}}{3}\right)\left(\frac{dx}{\cancel{x}}\right)$

$= \ln x\left(\frac{2x^3}{3}\right) - \int \frac{2x^2}{3} dx$

$= \ln x\left(\frac{2x^3}{3}\right) - \frac{2}{3}\int x^2 dx$

$= \ln x\left(\frac{2x^3}{3}\right) - \frac{2}{3}\int \frac{x^3}{3}$

$= \ln x\left(\frac{2x^3}{3}\right) - \frac{2x^3}{9} + C$

3. $x^2 \sin x \, dx$

Solution

$u = x^2, \ dv = \sin x \, dx$

$\frac{du}{dx} = 2x, \ v = -\cos x$

$\int u dv = uv - \int v du$

$= x^2(-\cos x) - \int -\cos x (2x) dx \quad -(i)$

$= x^2(-\cos x) - \cancel{2x \sin x} - \int 2 \sin x \, dx$

$= -x^2 \cos x - 2x \sin x - 2\cos x + C$

$\int -\cos x (2x) dx$

$u = 2x, \ dv = -\cos x \, dx$

$\frac{du}{dx} = 2, \ v = \sin x$

$\int -\cos x (2x) dx = (2x)(\sin x) - \int \sin x \, 2 dx$

$= 2x \sin x - \int 2 \sin x \, dx$

4. $x \cos x \, dx$

Solution

$u = x, \ dv = \cos x \, dx$

$\frac{du}{dx} = 1, \ v = \sin x$

$\int u dv = uv - \int v du$

$\int x \cos x \, dx = (x)(\sin x) - \int \sin x \, dx$

$= x \sin x - (-\cos x)$

$= x \sin x + \cos x + C$