

$$U_t - C U_{xx} = 0$$

$$\frac{du}{dt} - C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = \frac{C d^2u}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = C \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}] \quad r = \frac{C \Delta t}{\Delta x^2}, \quad C=1$$

$$U(x,0) = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \quad \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j} \quad \leftarrow *$$

when $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

when $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

$$\begin{aligned} \text{when } i=3, j=0 &= U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \\ U_{3,1} &= 0.2176 \end{aligned}$$

$$\begin{aligned} \text{when } i=4, j=0 &= U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \\ U_{4,1} &= 0.5648 \end{aligned}$$

for j=1.

$$\begin{aligned} \text{when } i=1 & \therefore U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1}) \\ &= 0.5U_{0,1} + 0.5U_{2,1} = 0 + 0.5(0.0656) \\ U_{1,2} &= 0.0328 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 & \therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ &= 0.5(0.0128) + 0.5(0.2176) \\ U_{2,2} &= 0.1152 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 & U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1}) \\ &= 0.5(0.0656) + 0.5(0.5648) \\ U_{3,2} &= 0.3152 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 & U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1}) \\ &= 0.5(0.2176) + 0.5(1) \\ U_{4,2} &= 0.6088 \end{aligned}$$

for j=2

when i=1

$$\begin{aligned} \therefore U_{1,3} &= 0.5(U_{0,2}) + 0.5(U_{2,2}) \\ &= 0 + 0.5(0.1152) = 0.0576 \end{aligned}$$

when i=2

$$\begin{aligned} U_{2,3} &= 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ &= 0.5(0.0328) + 0.5(0.3152) = 0.174 \end{aligned}$$

when i=3

$$\begin{aligned} U_{3,3} &= 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ &= 0.5(0.1152) + 0.5(0.6088) = 0.362 \end{aligned}$$

when $i = 1$

$$= 0.5(0.3152) + 0.5(1)$$

$$U_{4,3} = 0.6576$$

for $J = 3$

when $i = 1$

$$U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ = 0.5(0) + 0.5(0.174) = 0.087$$

when $i = 2$

$$U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ = 0.5(0.0576) + 0.5(0.362) = 0.2098$$

when $i = 3$

$$U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ = 0.5(0.174) + 0.5(0.6576) = 0.4158$$

when $i = 4$

$$U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ = 0.5(0.362) + 0.5(1) \\ = 0.681$$

for $J = 4$

when $i = 1$

$$U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ = 0.5(0.2098) = 0.1049$$

when $i = 2$

$$U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ = 0.5(0.087) + 0.5(0.4158) = 0.2514$$

when $i = 3$

$$U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ = 0.5(0.2098) + 0.5(0.681) \\ = 0.4454$$

when $i = 4$

$$U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ = 0.5(0.4158) + 0.5(1) \\ = 0.7079$$

Δt

0.1

0.08

0.06

0.04

0.02

0

Δx

i

| J Temp (K) | | | | | | | |
|---------------|---|--------|--------|--------|--------|---|--|
| 5 | 0 | 0.1049 | 0.2514 | 0.4454 | 0.7079 | 1 | |
| 4 | 0 | 0.087 | 0.2098 | 0.4158 | 0.681 | 1 | |
| 3 | 0 | 0.0576 | 0.174 | 0.362 | 0.6576 | 1 | |
| 2 | 0 | 0.0328 | 0.1152 | 0.3152 | 0.6008 | 1 | |
| 1 | 0 | 0.0128 | 0.0656 | 0.2176 | 0.5648 | 1 | |
| 0 | 0 | 0.0016 | 0.028 | 0.1296 | 0.4046 | 1 | |
| | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 | |
| | 0 | 1 | 2 | 3 | 4 | 5 | |