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17/ENG01/021
MECHANICAL ENGINEERING

$$U_t - (U_{xx} = 0$$

$$\frac{d_u}{dt} - \left(\frac{d^2 u}{dx^2} \right) = 0$$

$$\frac{d_u}{dt} = \left(\frac{d^2 u}{dx^2} \right)$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = \left(\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2} \right)$$

$$U_{i,j+1} - U_{i,j} = \left(\frac{\Delta t}{\Delta x^2} \right) [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}] \cdot \int_0^{\Delta t} \frac{d_u}{dx^2}, c=1$$

$$U(x,0) = x^4$$

$$\Delta x = 0.2m, \quad \Delta t = 0.02 \text{ day}$$

For initial conditions

$$\Delta t \ x = 0 \rightarrow x^4 = 0$$

$$\Delta t \ x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\Delta t \ x = 0.4 \rightarrow 0.4^4 = 0.0256$$

$$\Delta t \ x = 0.6 \rightarrow 0.6^4 = 0.1296$$

$$\Delta t \ x = 0.8 \rightarrow 0.8^4 = 0.4096$$

$$\Delta t \ x = 1 \rightarrow 1^4 = 1$$

$$U_{i,j+1} = U_{i,j} + r [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} = r U_{i-1,j} + (1-2r)U_{i,j} + r U_{i+1,j}$$

When $i=1, j=0$

$$U_{1,1} = 0.5U_{0,0} + 0.5U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

When $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,0} = 0.656$$

$$\begin{aligned} \text{When } i=3, j=0: U_{3,0} &= 0.5(U_{2,0}) + 0.5(U_{1,0}) \\ &= 0.5(0.656) + 0.5(0.696) \\ U_{3,0} &= 0.676 \end{aligned}$$

$$\begin{aligned} \text{When } i=4, j=0: U_{4,0} &= 0.5(U_{3,0}) + 0.5(U_{5,0}) \\ &= 0.5(0.676) + 0.5(0.608) \\ U_{4,0} &= 0.642 \end{aligned}$$

For j=1:

$$\begin{aligned} \text{When } i=1: U_{1,1} &= 0.5(U_{0,1}) + 0.5(U_{2,1}) \\ &= 0.5(U_{0,1}) + 0.5(U_{2,1}) = 0 + 0.3(0.068) \\ U_{1,1} &= 0.0328 \end{aligned}$$

$$\begin{aligned} \text{When } i=2: U_{2,1} &= 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ &= 0.5(0.0328) + 0.5(0.2176) \\ U_{2,1} &= 0.1252 \end{aligned}$$

$$\begin{aligned} \text{When } i=3: U_{3,1} &= 0.5(U_{2,1}) + 0.5(U_{4,1}) \\ &= 0.5(0.1252) + 0.5(0.5648) \\ U_{3,1} &= 0.345 \end{aligned}$$

$$\begin{aligned} \text{When } i=4: U_{4,1} &= 0.5(U_{3,1}) + 0.5(U_{5,1}) \\ &= 0.5(0.345) + 0.5(1) \\ U_{4,1} &= 0.6725 \end{aligned}$$

For j=2:

$$\begin{aligned} \text{When } i=1: U_{1,2} &= 0.5(U_{0,2}) + 0.5(U_{2,2}) \\ &= 0 + 0.5(0.1152) = 0.0576 \end{aligned}$$

$$\begin{aligned} \text{When } i=2 \quad U_{2,3} &= 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ &= 0.5(0.0528) + 0.5(0.3152) = 0.174 \end{aligned}$$

$$\begin{aligned} \text{When } i=3 \quad U_{3,3} &= 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ &= 0.5(0.1152) + 0.5(0.608) = 0.362 \end{aligned}$$

$$\begin{aligned} \text{When } i=4 \quad U_{4,3} &= 0.5(0.3152) + 0.5(1) \\ &= 0.6576 \end{aligned}$$

For $J=3$

$$\begin{aligned} \text{When } i=1 \quad U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.174) = 0.087 \end{aligned}$$

$$\begin{aligned} \text{When } i=2 \quad U_{2,4} &= 0.5((,3) + 0.5)(U_{3,3}) \\ &= 0.5(0.0576 + 0.5(0.362)) = 0.2098 \end{aligned}$$

$$\begin{aligned} \text{When } i=3 \quad U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.174) + 0.5(0.6576) = 0.4158 \end{aligned}$$

$$\begin{aligned} \text{When } i=4 \quad U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681 \end{aligned}$$

For $J=4$

$$\begin{aligned} \text{When } i=1 \quad U_{1,5} &= 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ &= 0.5(0.2098) = 0.1049 \end{aligned}$$

$$\begin{aligned} \text{When } i=2 \quad U_{2,5} &= 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ &= 0.5(0.087) + 0.5(0.4158) = 0.2514 \end{aligned}$$

When $i = 3$ $U_{3.5} = 0.5(U_{2.4}) + 0.5(U_{4.4})$
 $= 0.5(0.2098) + 0.5(0.681)$
 $= 0.4454$

When $i = 4$ $U_{4.5} = 0.5(U_{3.4}) + 0.5(U_{5.4})$
 $= 0.5(0.4158) + 0.5(1)$
 $= 0.7079$

ΔL	$\frac{T}{TEMP(K)}$							
0.1	5	0	0.1049	0.2514	0.4454	0.7079	1	
0.08	4	0	0.087	0.2098	0.6158	0.681	1	
0.06	3	0	0.0576	0.174	0.362	0.6576	1	
0.04	2	0	0.0328	0.1152	0.3152	0.6008	1	
0.02	1	0	0.0128	0.0636	0.2176	0.5648	1	
0	0	0	0.0016	0.028	0.1296	0.4096	1	
Δx		0	0.2	0.4	0.6	0.8	1	
	i	0	1	2	3	4	5	