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18/eng 03/059

Civil Engineering ENG3826

$$U_t - CU_{xx} = 0$$

$$\frac{dU}{dt} - C \frac{d^2U}{dx^2} = 0$$

$$\frac{dU}{dt} = C \frac{d^2U}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = C \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta x^2}$$

$$U_{i,j+1} - U_{i,j} = C \frac{\Delta t}{\Delta x^2} [U_{i,j+1} - 2U_{i,j} + U_{i,j-1}] - \sqrt{C} \frac{\Delta t}{\Delta x} \quad , l=1$$

$$\Delta x = 0.2 \text{ m} \quad \Delta t = 0.02 \text{ day}$$

For Initial Conditions

$$\text{At } x=0 \rightarrow x^4 = 0$$

$$\text{At } x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\text{At } x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{ij+1} = U_{ij} + r[U_{i+1,j} - 2U_{ij} + U_{i-1,j}]$$

$$U_{ij+1} = r U_{i+1,j} + (1-2r)U_{ij} + r U_{i-1,j}$$

When $i=1, j=0$

$$\begin{aligned} U_{1,1} &= 0.5 U_{0,0} + 0.5 U_{2,0} \\ &= 0.5(0) + 0.5(0.0256) \\ U_{1,1} &= 0.0128 \end{aligned}$$

When $i=2, j=0$

$$\begin{aligned} U_{2,1} &= 0.5(U_{1,0}) + 0.5(U_{3,0}) \\ &= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296) \\ U_{2,1} &= 0.0656 \end{aligned}$$

When $i=3, j=0$

$$\begin{aligned} U_{3,1} &= 0.5(U_{2,0}) + 0.5(U_{4,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \\ U_{3,1} &= 0.2176 \end{aligned}$$

When $i=4, j=0$

$$\begin{aligned} U_{4,1} &= 0.5(U_{3,0}) + 0.5(U_{5,0}) \\ &= 0.5(0.0256) + 0.5(0.4096) \\ U_{4,1} &= 0.5648 \end{aligned}$$

For $j=1$

When $i=1$

$$\begin{aligned} U_{1,2} &= 0.5(U_{0,1}) + 0.5(U_{2,1}) \\ &= 0.5 U_{0,1} + 0.5(0.0656) = 0 + 0.5(0.0656) \\ U_{1,2} &= 0.0328 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 2 & :- U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1}) \\ & = 0.5(0.0128) + 0.5(0.2176) \\ U_{2,2} & = 0.1152 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 3 & \quad U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1}) \\ & \quad 0.5(0.0656) + 0.5(0.5648) \\ U_{3,2} & = 0.3152 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 4 & \quad U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1}) \\ & = 0.5(0.2176) + 0.5(1) \\ U_{4,2} & = 0.6088 \end{aligned}$$

For $\bar{j} = 2$

$$\begin{aligned} \text{When } \bar{l} = 1 & :- U_{1,3} = 0.5(U_{0,1}) + 0.5(U_{2,2}) \\ & = 0 + 0.5(0.1152) = 0.0576 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 2 & \quad U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2}) \\ & = 0.5(0.0328) + 0.5(0.3152) = \\ & = 0.174 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 3 & \quad U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2}) \\ & = 0.5(0.1152) + 0.5(0.6088) \\ & = 0.362 \end{aligned}$$

$$\begin{aligned} \text{When } \bar{l} = 4 & \quad U_{4,3} = 0.5 U \\ & = 0.5(0.3152) + 0.5(1) \\ U_{4,3} & = 0.6576 \end{aligned}$$

For $J=3$

$$\begin{aligned}\text{When } \bar{i}=1 \quad U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.174) = 0.087\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=2 \quad U_{2,4} &= 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ &= 0.5(0.0576) + 0.5(0.362) = 0.2098\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=3 \quad U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.174) + 0.5(0.6576) = 0.4158\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=4 \quad U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681\end{aligned}$$

For $J=4$

$$\begin{aligned}\text{When } \bar{i}=1 \quad U_{1,5} &= 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ &= 0.5(0) + 0.5(0.2098) = 0.1049\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=2 \quad U_{2,5} &= 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ &= 0.5(0.087) + 0.5(0.4158) = 0.2514\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=3 \quad U_{3,5} &= 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ &= 0.5(0.2098) + 0.5(0.681) \\ &= 0.4454\end{aligned}$$

$$\begin{aligned}\text{When } \bar{i}=4 \quad U_{4,5} &= 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079\end{aligned}$$

Δt	Temp (K)						
0-1	5	0	0-1049	0-2514	0-4454	0-7079	1
0-08	4	0	0-087	0-2098	0-4158	0-681	1
0-06	3	0	0-0576	0-174	0-362	0-6576	1
0-04	2	0	0-0328	0-1152	0-3152	0-6008	1
0-02	1	0	0-0128	0-0656	0-2176	0-5648	1
0	0	0	0-0016	0-028	0-1296	0-4096	1
Δx		0	0-2	0-4	0-6	0-8	1
	6	0	1	2	3	4	5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1

0.5

