

ADEPOJU MARY ABIMBOLA

17/ENG031004

CIVIL ENGINEERING

ENG 382

ENGINEERING MATHEMATICS II

ASSIGNMENT

$$u_t - C u_{xx} = 0$$

$$\frac{du}{dt} - C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u(x, 0) = x^4$$

where $\Delta x = 0.2 \text{ m}$, $\Delta t = 0.02 \text{ day}$

initial conditions

$$\text{At } x = 0 = 0^4 = 0$$

$$\text{At } x = 0.2 = 0.2^4 = 0.0016$$

$$At \quad x = 0.4 = 0.4^4 = 0.0256$$

$$At \quad x = 0.6 = 0.6^4 = 0.1296$$

$$At \quad x = 0.8 = 0.8^4 = 0.4096$$

$$At \quad x = 1 = 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + \delta [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} = \delta u_{i-1,j} + (1-2\delta) u_{i,j} + \delta u_{i+1,j} \dots \text{eq. (1)}$$

When $i=1$ & $j=0$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$

$$= (0.5 \times 0) + (0.5 \times 0.0256)$$

$$u_{1,1} = 0.0128$$

When $i=2$, $j=0$

$$u_{2,1} = 0.5 (u_{1,0}) + 0.5 (u_{3,0})$$

$$= 0.5 (0.0016) + 0.5 (0.1296)$$

$$u_{2,1} = 0.0656$$

When $i=3$, $j=0$

$$u_{3,1} = 0.5 (u_{2,0}) + 0.5 (u_{4,0})$$

$$= 0.5 (0.0256) + 0.5 (0.4096)$$

$$u_{3,1} = 0.2176$$

When $i=4$, $j=0$

$$u_{4,1} = 0.5 (u_{3,0}) + 0.5 (u_{5,0})$$

$$= 0.5 (0.1296) + 0.5 (1)$$

$$u_{4,1} = 0.5647$$

When $i=1$ & $j=1$

$$u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1})$$

$$u_{1,2} = 0.5(0) + 0.5(0.0666)$$

$$u_{1,2} = 0.0328$$

When $i=2$ & $j=1$

$$u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$$

$$= 0.5(0.0128) + (0.5 \times 0.2176)$$

$$u_{2,2} = 0.1152$$

When $i=3$ & $j=1$

$$u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1})$$

$$= 0.5(0.0666) + 0.5(0.6647)$$

$$u_{3,2} = 0.3152$$

When $i=4$ & $j=1$

$$u_{4,2} = 0.5(u_{3,1}) + 0.5(u_{5,1})$$

$$= 0.5(0.2176) + 0.5(1)$$

$$u_{4,2} = 0.6088$$

When $i=1$ & $j=2$

$$u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2})$$

$$= 0.5(0) + 0.5(0.1152)$$

$$u_{1,3} = 0.0576$$

When $i=2$ & $j=2$

$$u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2})$$

$$= 0.5(0.0328) + 0.5(0.3152)$$

$$u_{2,3} = 0.174$$

When $i = 3, j = 2$

$$\begin{aligned}u_{3,3} &= 0.5(u_{2,2}) + 0.5(u_{4,2}) \\ &= 0.5(0.1152) + 0.5(0.6088) \\ u_{3,3} &= 0.362\end{aligned}$$

When $i = 4, j = 3$

$$\begin{aligned}u_{4,3} &= 0.5(u_{3,2}) + 0.5(u_{5,2}) \\ &= 0.5(0.3152) + (0.5 \times 1) \\ u_{4,3} &= 0.6576\end{aligned}$$

When $i = 1, j = 3$

$$\begin{aligned}u_{1,4} &= 0.5(u_{0,3}) + 0.5(u_{2,3}) \\ &= 0.5(0) + 0.5(0.174) \\ u_{1,4} &= 0.087\end{aligned}$$

When $i = 2, j = 3$

$$\begin{aligned}u_{2,4} &= 0.5(u_{1,3}) + 0.5(u_{3,3}) \\ &= 0.5(0.0576) + 0.5(0.362) \\ u_{2,4} &= 0.2098\end{aligned}$$

When $i = 3, j = 3$

$$\begin{aligned}u_{3,4} &= 0.5(u_{2,3}) + 0.5(u_{4,3}) \\ &= 0.5(0.174) + (0.5 \times 0.6576) \\ u_{3,4} &= 0.4768\end{aligned}$$

When $i = 4, j = 3$

$$\begin{aligned}u_{4,4} &= 0.5(u_{3,3}) + 0.5(u_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ u_{4,4} &= 0.681\end{aligned}$$

When $i=1$ & $j=4$

$$u_{1,5} = 0.5(u_{0,4}) + 0.5(u_{2,4})$$

$$= 0.5(0) + 0.5(0.2098)$$

$$u_{1,5} = 0.1049$$

When $i=2$ & $j=4$

$$u_{2,5} = 0.5(u_{1,4}) + 0.5(u_{3,4})$$

$$= 0.5(0.087) + 0.5(0.4158)$$

$$u_{2,5} = 0.2514$$

When $i=3$ & $j=4$

$$u_{3,5} = 0.5(u_{2,4}) + 0.5(u_{4,4})$$

$$= 0.5(0.2098) + 0.5(0.687)$$

$$u_{3,5} = 0.4454$$

When $i=4$ & $j=4$

$$u_{4,5} = 0.5(u_{3,4}) + 0.5(u_{5,4})$$

$$= 0.5(0.4158) + 0.5(1)$$

$$u_{4,5} = 0.7079$$

$j \backslash i$ Series (K)	0	1	2	3	4	5
5	0	0.1049	0.2514	0.4454	0.7079	1
4	0	0.087	0.2098	0.4158	0.687	1
3	0	0.0516	0.114	0.362	0.6516	1
2	0	0.0328	0.1152	0.3152	0.6088	1
1	0	0.0128	0.0656	0.2176	0.5647	1
0	0	0.0016	0.024	0.1296	0.4096	1
0	0	0.2	0.4	0.6	0.8	1
i	0	1	2	3	4	5

