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17/Eng03/048
Civil Engineering
Assignment 6.

$$u_t - c u_{xx} = 0.$$

$$\frac{du}{dt} - c\frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = c\frac{d^2u_i}{dx^2}$$

$$\frac{u_{ij+1} - u_{ij}}{\Delta t} = c\left(\frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{\Delta x^2}\right)$$

$$u_{ij+1} - u_{ij} = \frac{c\Delta t}{\Delta x^2}\left[u_{i+1,j} - 2u_{ij} + u_{i-1,j}\right]$$

$$U[x,0] = x^4 k.$$

$\Delta x = 0.2m$, $\Delta t = 0.02$ day.

for initial condition

At $x = 0 \rightarrow x^4 = 0.$

At $x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$

At $x = 0.4 \rightarrow 0.4^4 = 0.256$

At $x = 0.6 \rightarrow 0.6^4 = 0.1296$

At $x = 0.8 \rightarrow 0.8^4 = 0.4096$

At $x = 1 \rightarrow 1^4 = 1.$

$$\therefore u_{ij+1} = u_{ij} + r\left[u_{i+1,j} - 2u_{ij} + u_{i-1,j}\right]$$

$$u_{ij+1} = ru_{i-1,j} + (1-2r)u_{ij} + ru_{i+1,j}$$

when $i = 1, j = 0.$

$$u_{1,1} = 0.5u_{0,0} + 0.5u_{2,0}.$$

$$= 0.5(0) + 0.5(0.0256)$$

$$u_{01} = 0.0128.$$

$U_{2,1} = 0.5(U_{1,0}) + 0.5[U_{3,0})$.

$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$

$U_{2,1} = 0.0656$.

when $i = 3, j = 0$.

$U_{3,1} = .5\ (U_{2,0}) + 0.5\ (U_{4,0})$

$= 0.5\ (0.0256) + 0.5\ (0.4096)$.

$U_{3,1} = 0.2176$.

when $i = 4; j = 0$ - $U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$.

$= 0.5(0.0256) + 0.5(0.4096)$.

$\therefore U_{4,1} = 0.5648$.

for 1

$\therefore j = 1$

when $i = 1$ $U_{1,2} = 0.5(U_{0,2}) + 0.5(U_{2,1})$.

$= 0.5\ U_{0,1} + 0.5 U_{2,1} = 0 \cdot 0.5\ (0.0656)$

$U_{1,2} = 0.0328$.

when $i = 2$ $\therefore U_{2,2} = 0.5\ (U_{1,1}) + 0.5\ (U_{3,1})$.

$= 0.5\ [0.0128 + 0.5\ (0.2176)$

$U_{2,2} = 0.1152$

when $i = 3$

$U_{3,2} = 0.5\ (U_{0,1}) + 0.5\ (U_{4,1})$

$= 0.5\ (0.0896) + 0.5\ [0.5648)$

$U_{3,2} = 0.3152$

when $i = 4$, $U_{4,2} = 0.5\ (U_{3,1}) + 0.5\ (U_{5,1})$

$= 0.5\ (0.2176) + 0.5\ (1)$.

$U_{4,2} = 0.6088$

for $j = 2$

when $i = 1$ $\therefore U_{1,3} = 0.5\ (U_{0,2}) + 0.5\ (U_{2,2})$

$= 0 + 0.5\ (0.1152) = 0.0576$

where i = 1 ∴ $U_{1,3} = 0.5(U_{0,2}) + 0.5(U_{2,2})$.

$= 0 + 0.5(0.1152)$

$= 0.0576$.

when i = 2 $U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2})$.

$= 0.5(0.0328) + 0.5(0.3152)$

$= 0.174$.

when i = 3 $U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2})$

$= 0.5(0.1152) + 0.5(0.6088) = 0.362$.

when i = 4 $U_{4,3} = 0.5(U_{3,2}) + 0.5(U_{5,2})$.

$= 0.5(0.3152) + 0.5(1)$.

$U_{4,3} = 0.6576$.

for J3.

when i = 1

$U_{1,4} = 0.5(0,3) + 0.5(U_{2,3})$.

$= 0.5(0) + 0.5(0.174)$.

$= 0.084$.

when i = 2

$U_{2,4} = 0.5(1,3 + 0.5(U_{3,3})$

$= 0.5(0.0576) + 0.5(0.362) = 0.2098$

when i = 3

$U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3})$

$= 0.5(0.174) + 0.5(0.6576$

$= 0.4158$

when i = 4 $U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3})$

$= 0.5(0.362) + 0.5(1)$

$= 0.681$.

for J = 4

when i = 1 $= U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4}$

$= 0.5\ (0.2098) = 0.1049$

when $i = 2$ $\quad U_{2,5} = 0.5\ (U_{1,4}) + 0.5\ (U_{3,4})$.

$= 0.5\ (0.087) + 0.5\ (0.4158) = 0.2514$

when $i = 3$

$U_{3,5} = 0.5\ (U_{2,4}) + 0.5\ (U_{4,4})$.

$0.5\ (0.2098) + 0.5\ (0.681)$.

$= 0.4454$

when $i = 4$ $\quad U_{4,5} = 0.5\ (U_{3,4}) + 0.5\ (U_{5,4})$.

$= 0.5\ (0.4158) + 0.5\ (1)$.

$= 0.7079$.

Δt	j / x (U_{ij})						
0.1	5	0	0.1049	0.2514	0.4449	0.7079	1
0.08	4	0	0.087	0.2098	0.4158	0.681	1
0.06	3	0	0.0576	0.174	0.362	0.6596	1
0.04	2	0	0.0328	0.1154	0.3152	0.6008	1
0.02	1	0	0.0128	0.0656	0.206	0.5647	1
0	0	0	0.0016	0.0256	0.1296	0.4096	1
Δx		0	0.2	0.4	0.6	0.8	1
	i	0	1	2	3	4	5

0.5

0.1	0	0.1049	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.0576	0.174	0.362	0.6576	1
0.04	0	0.0328	0.1152	0.3152	0.6088	1
0.02	0	0.0128	0.0656	0.2176	0.5648	1
0	0	0.0016	0.0256	0.1296	0.4096	1
	0	0.2	0.4	0.6	0.8	1