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17/ENG03/046
Civil Engineering

$$U_t - C U_{xx} = 0$$

$$\frac{du}{dt} - C\frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C\frac{d^2u}{dx^2}$$

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta t} = \frac{C(U_{i+1,j} - 2U_{i,j} + U_{i-1,j})}{\Delta x}$$

$$U_{i,j+1} - U_{i,j} = \frac{C\Delta t}{\Delta x^2}\left[U_{i+1,j} - 2U_{i,j} + U_{i-1,j}\right] \cdot \text{Fn} \quad \Delta t^2/\Delta x^2, \; C=1$$

$$U(x,0) = x^4 h$$

$\Delta x = 0.2m$, $\Delta t = 0.02$ hrs

for limited conditions

At $x=0 \rightarrow x^4 = 0$

At $x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$

At $x = 0.4 \rightarrow 0.4^4 = 0.0256$

At $x = 0.6 \rightarrow 0.6^4 = 0.1296$

At $x = 0.8 \rightarrow 0.8^4 = 0.4096$

At $x = 1 \rightarrow 1^4 = 1$

$$\therefore U_{i,j+1} = U_{i,j} + r\left[U_{i+1,j} - 2U_{i,j} + U_{i-1,j}\right]$$
$$= rU_{i-1,j} + (1-2r)U_{i,j} + rU_{i+1,j}$$

when $i=1, j=0$

$$U_{1,1} = 0.5\,U_{0,0} + 0.5\,U_{2,0}$$
$$= 0.5(0) + 0.5(0.0256)$$
$$= 0.0128$$

when $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{0,2})$$
$$= 0.5(0) + 0.5(0.0256) \quad 0.5(1.6\times10^{-3}) + 0.5(0.1296)$$
$$= 0.0656$$

when $i=3, j=0$,

$$u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{1,0})$$
$$= 0.5(0.0256) + 0.5(0.4096)$$
$$= 0.2176$$

When $i=4, j=0$;

$$u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0})$$
$$= 0.5(0.0256) + 0.5(0.4096)$$
$$= 0.5648$$

For $j=1$

when $i=1$; $u_{1,2} = 0.5(u_{0,2}) + 0.5(u_{2,1})$
$$= 0 + 0.5(0.0656)$$
$$= 0.0328$$

when $i=2$; $u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$
$$= 0.5(0.0124) + 0.5(0.2176)$$
$$= 0.1152$$

$i=3$; $u_{3,2} = 0.5(u_{2,1}) + 0.5(u_{4,1})$
$$= 0.5(0.0656) + 0.5(0.5648)$$
$$= 0.3152$$

$i=4$; $u_{4,2} = 0.5(u_{3,1}) + 0.5(u_{5,1})$
$$= 0.5(0.2176) + 0.5(0)$$
$$= 0.0088$$

j = 2;

i = 1 ; $U_{1,2} = 0.5(U_{0,2}) + 0.5(U_{2,2})$

$= 0 + 0.5(0.1152) = 0.0576$

i = 2 ; $U_{2,2} = 0.5(U_{1,2}) + 0.5(U_{3,2})$

$= 0.5(0.0324) + 0.5(0.3152) = 0.174$

i = 3 ; $U_{3,2} = 0.5(U_{2,2}) + 0.5(U_{4,2})$

$= 0.5(0.1152) + 0.5(0.6048) = 0.362$

i = 4 ; $U_{4,2} = 0.5(U_{3,2}) + 0.5(U_{1,2})$

$= 0.5(0.3152) + 0.5(1) = 0.6576$

For j = 3

i = 1 ; $U_{1,3} = 0.5(U_{0,3}) + 0.5(U_{2,3})$

$= 0.5(0) + 0.5(0.174) = 0.044$

i = 2 ; $U_{2,3} = 0.5(U_{1,3}) + 0.5(U_{3,3})$

$= 0.5(0.0576) + 0.5(0.312) = 0.2698$

i = 3 ; $U_{3,3} = 0.5(U_{2,3}) + 0.5(U_{4,3})$

$= 0.5(0.174) + 0.5(0.6576) = 0.4158$

i = 4 ; $U_{4,3} = 0.5(U_{3,3}) + 0.5(U_{5,3})$

$= 0.5(0.362) + 0.5(1) = 0.681$

For j = 4

i = 1 ; $U_{1,4} = 0.5(U_{0,4}) + 0.5(U_{2,4})$

$= 0.5(0.2094) + 0.5(0.1049) = 0.1047$

i = 2 ; $U_{2,4} = 0.5(U_{1,4}) + 0.5(U_{3,4})$

$= 0.5(0.047) + 0.5(0.4158) = 0.2514$

i = 3 ; $U_{3,4} = 0.5(U_{2,4}) + 0.5(U_{4,4})$

$= 0.5(0.2094) + 0.5(0.681) = 0.4454$

i = 4 ; $U_{4,4} = 0.5(U_{3,4}) + 0.5(U_{5,4})$

$= 0.5(0.4454) + 0.5(1) = 0.7074$

| del(t) | J/Temp(k) | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0 | 0.1049 | 0.2514 | 0.4454 | 0.7079 | 1 |
| 0.08 | 0.08 | 0 | 0.087 | 0.2098 | 0.4158 | 0.681 | 1 |
| 0.06 | 0.06 | 0 | 0.0576 | 0.174 | 0.362 | 0.6576 | 1 |
| 0.04 | 0.04 | 0 | 0.0328 | 0.1152 | 0.3152 | 0.6088 | 1 |
| 0.02 | 0.02 | 0 | 0.0128 | 0.0656 | 0.2176 | 0.5648 | 1 |
| 0 | 0 | 0 | 0.0016 | 0.0256 | 0.1296 | 0.4096 | 1 |
| del(x) | | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
| i | | 0 | 1 | 2 | 3 | 4 | 5 |