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Question

**(1).**Analyze linear transformation giving **5 examples each.**

**Definition**

A ***linear transformation*** is a transformation T:Rn→Rm satisfying

T(u+v)=T(u)+T(v)T(cu)=cT(u)

for all vectors u,v in Rn and all scalars c.

Let T:Rn→Rm be a matrix transformation: T(x)=Ax for an m×n matrix A. By this [proposition in Section 2.3](https://textbooks.math.gatech.edu/ila/linear-transformations.html), we have

T(u+v)=A(u+v)=Au+Av=T(u)+T(v)T(cu)=A(cu)=cAu=cT(u)

for all vectors u,v in Rn and all scalars c. Since a matrix transformation satisfies the two defining properties, it is a linear transformation

We will see in the next [subsection](https://textbooks.math.gatech.edu/ila/linear-transformations.html#linear-trans-is-matrix-trans) that the opposite is true: every linear transformation is a matrix transformation; we just haven't computed its matrix yet.

**Facts about linear transformations**

*Let T:Rn→Rm be a linear transformation. Then:*

1. *T(0)=0.*
2. *For any vectors v1,v2,...,vk in Rn and scalars c1,c2,...,ck, we have*

*TAc1v1+c2v2+···+ckvkB=c1T(v1)+c2T(v2)+···+ckT(vk).*

*[Proof](https://textbooks.math.gatech.edu/ila/linear-transformations.html)*

In engineering, the second fact is called the *superposition principle*; it should remind you of the distributive property. For example, T(cu+dv)=cT(u)+dT(v) for any vectors u,v and any scalars c,d. To restate the first fact:

A linear transformation necessarily takes the zero vector to the zero vector.

**[Example(A non-linear transformation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.4%20A%20non-linear%20transformation)**

**[Example(Verifying linearity: dilation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.5%20Verifying%20linearity%3A%20dilation)**

**[Example(Verifying linearity: rotation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.6%20Verifying%20linearity%3A%20rotation)**

**[Example(A transformation defined by a formula)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.7%20A%20transformation%20defined%20by%20a%20formula)**

One can show that, if a transformation is defined by formulas in the coordinates as in the above example, then the transformation is linear if and only if each coordinate is a linear expression in the variables with no constant term.

**[Example(A translation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.8%20A%20translation)**

**[Example(More non-linear transformations)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.9%20More%20non-linear%20transformations)**

When deciding whether a transformation T is linear, generally the first thing to do is to check whether T(0)=0; if not, T is automatically not linear. Note however that the non-linear transformations T1 and T2 of the above example do take the zero vector to the zero vector.

**[Challenge](https://textbooks.math.gatech.edu/ila/linear-transformations.html)**

**The Standard Coordinate Vectors**

In the next subsection, we will present the relationship between linear transformations and matrix transformations. Before doing so, we need the following important notation.

**Standard coordinate vectors**

The ***standard coordinate vectors in Rn*** are the n vectors

e1=EIIIIG10...00FJJJJH,e2=EIIIIG01...00FJJJJH,...,en−1=EIIIIG00...10FJJJJH,en=EIIIIG00...01FJJJJH.

The ith entry of ei is equal to 1, and the other entries are zero.

*From now on,* for the rest of the book, we will use the symbols e1,e2,... to denote the standard coordinate vectors.

There is an ambiguity in this notation: one has to know from context that e1 is meant to have n entries. That is, the vectors

K10LandC100D

may both be denoted e1, depending on whether we are discussing vectors in R2 or in R3.

The standard coordinate vectors in R2 and R3 are pictured below.

e1e2inR2inR3e1e2e3

These are the vectors of length 1 that point in the positive directions of each of the axes.

**Multiplying a matrix by the standard coordinate vectors**

*If A is an m×n matrix with columns v1,v2,...,vm, then Aei=vi for each i=1,2,...,n:*

C|||v1v2···vn|||Dei=vi.

*In other words, multiplying a matrix by ei simply selects its ith column.*

For example,

C123456789DC100D=C147DC123456789DC010D=C258DC123456789DC001D=C369D.

**Definition**

The ***n×n identity matrix*** is the matrix In whose columns are the n standard coordinate vectors in Rn:

In=EIIIIG10···0001···00...............00···1000···01FJJJJH.

We will see in this [example](https://textbooks.math.gatech.edu/ila/linear-transformations.html) below that the identity matrix is the matrix of the [identity transformation](https://textbooks.math.gatech.edu/ila/linear-transformations.html).

**The Matrix of a Linear Transformation**

Now we can prove that every linear transformation is a matrix transformation, and we will show how to compute the matrix.

**Theorem(The matrix of a linear transformation)**

*Let T:Rn→Rm be a linear transformation. Let A be the m×n matrix*

A=C|||T(e1)T(e2)···T(en)|||D.

*Then T is the matrix transformation associated with A: that is, T(x)=Ax.*

*[Proof](https://textbooks.math.gatech.edu/ila/linear-transformations.html)*

The matrix A in the above theorem is called the ***standard matrix*** for T. The columns of A are the vectors obtained by evaluating T on the n standard coordinate vectors in Rn. To summarize part of the theorem:

Matrix transformations are the same as linear transformations.

**Dictionary**

Linear transformations are the same as matrix transformations, which come from matrices. The correspondence can be summarized in the following dictionary.

T:Rn→RmLineartransformation−−−→m×nmatrixA=C|||T(e1)T(e2)···T(en)|||DT:Rn→RmT(x)=Ax←−−−m×nmatrixA

**[Example(The matrix of a dilation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.17%20The%20matrix%20of%20a%20dilation)**

**[Example(The matrix of a rotation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.18%20The%20matrix%20of%20a%20rotation)**

We saw in the above example that the matrix for counterclockwise rotation of the plane by an angle of θ is

A=Kcosθ−sinθsinθcosθL.

**[Example(A transformation defined by a formula)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.19%20A%20transformation%20defined%20by%20a%20formula)**

**[Example(A transformation defined in steps)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.20%20A%20transformation%20defined%20in%20steps)**

Recall from this [definition in Section 3.1](https://textbooks.math.gatech.edu/ila/linear-transformations.html) that the *identity transformation* is the transformation IdRn:Rn→Rn defined by IdRn(x)=x for every vector x.

**[Example(The standard matrix of the identity transformation)](https://textbooks.math.gatech.edu/ila/linear-transformations.html%22%20%5Co%20%22Example%203.3.22%20The%20standard%20matrix%20of%20the%20identity%20transformation)**

We computed in this [example](https://textbooks.math.gatech.edu/ila/linear-transformations.html) that the matrix of the identity transform is the identity matrix: for every x in Rn,

x=IdRn(x)=Inx.

Therefore, Inx=x for all vectors x: the product of the identity matrix and a vector is the same vector.