

Emerging Clinton  
171ENG031021

Civil Engineering ENGG 382 6

$$u_t - C u_{xx} = 0$$

$$\frac{du}{dt} = \frac{C d^2 u}{dx^2} = 0$$

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$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = C \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = \frac{C \Delta t}{\Delta x^2} (u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$\cdot r = \frac{C \Delta t}{\Delta x^2}, r = 1$

$$u(x_0) = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

(for limited condition)

$$\Delta t \ x = 0 \rightarrow x^4 = 0$$

$$\Delta t \ x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\Delta t \ x = 0.4 \rightarrow 0.4^4 = 0.0256$$

$$\Delta t \ x = 0.6 \rightarrow 0.6^4 = 0.1296$$

$$\Delta t \ x = 0.8 \rightarrow 0.8^4 = 0.4096$$

$$\Delta t \ x = 1 \rightarrow 1^4 = 1$$

$$\therefore u_{i,j+1} = u_{i,j} + r (u_{i+1,j} - 2u_{i,j} + u_{i-1,j})$$

$$\text{When } i=1, j=0 \Rightarrow u_{1,1} = \frac{1}{2} u_{0,0} + (1-r) u_{1,0} + r u_{2,0}$$

$$u_{1,1} = 0.5 u_{0,0} + 0.5 u_{2,0}$$
$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

$$\text{When } i=2, j=0$$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$



$$\text{when } i=3, j=0 \Rightarrow U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

$$\text{when } i=4, j=0 \Rightarrow U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

for  $j=1$   
when  $L=1$

$$U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$$

$$= 0.5U_{0,1} + 0.5U_{2,1} = 0 + 0.5(0.068)$$

$$U_{1,2} = 0.0328$$

$$\text{when } i=2 \therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$$

$$= 0.5(0.068) + 0.5(0.2176)$$

$$U_{2,2} = 0.152$$

$$\text{when } i=3 \quad U_{3,2} = 0.5(U_{2,2}) + 0.5(U_{4,2})$$

$$= 0.5(0)$$



$$\text{when } \bar{i} = 3 \quad U_{3,2} = 0.5(U_{0,1}) + 0.5(U_{4,1}) \\ = 0.5(0.0656) + 0.5(0.5648)$$

$$U_{3,2} = 0.3152$$

$$\text{when } \bar{i} = 4 \quad U_{4,2} = 0.5(U_{2,1}) + 0.5(U_{5,1})$$

$$= 0.5(0.2176) + 0.5(1)$$

$$U_{4,2} = 0.6088$$

for  $J = 2$

when  $\bar{i} = 1$

$$U_{1,3} = 0.5(U_{0,1}) + 0.5(U_{2,1})$$

$$= 0 + 0.5(0.1152) = 0.0576$$

when  $\bar{i} = 2$

$$U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2})$$

$$= 0.5(0.0328) + 0.5(0.3152) = 0.174$$

when  $\bar{i} = 3$

$$U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2})$$

$$= 0.5(0.1152) + 0.5(0.6088) = 0.362$$



$$U_{t,3} = 0.5(0.515) + 0.5(0.6576)$$

For J=3

when  $i=1$

$$U_{1,4} = 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ = 0.5(0) + 0.5(0.174) = 0.087$$

when  $i=2$

$$U_{2,4} = 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ = 0.5(0.0576) + 0.5(0.362) = 0.2098$$

when  $i=3$

$$U_{3,4} = 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ = 0.5(0.174) + 0.5(0.6576) = 0.4158$$

when  $i=4$

$$U_{4,4} = 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ = 0.5(0.362) + 0.5(0.6576) = 0.5098$$

For J=4

when  $i=1$

$$U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ = 0.5(0.087) + 0.5(0.2098) = 0.1484$$

when  $i=2$

$$U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ = 0.5(0.087) + 0.5(0.4158) = 0.2518$$

when  $i=3$

$$U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ = 0.5(0.2098) + 0.5(0.5098) = 0.3598$$

when  $i=4$

$$U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ = 0.5(0.4158) + 0.5(1) = 0.7079$$



J \ Temp (K)						
5	0	0.1049	0.2514	0.4454	0.7079	1
4	0	0.087	0.2098	0.4158	0.681	1
3	0	0.0576	0.174	0.362	0.6576	1
2	0	0.0328	0.1152	0.3152	0.6008	1
1	0	0.0128	0.0656	0.2176	0.5698	1
0	0	0.0016	0.0288	0.1296	0.4046	1
	0	0.2	0.4	0.6	0.8	1
i	0	1	2	3	4	5

