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MATRIC NUMBER: 17/ENG03/059

DEPARTMENT: CIVIL ENGINEERING [ENG382]

ASSIGNMENT TITLE: ENG382 ASSIGNMENT IV

$$ut - C u_{xx} = 0$$

$$\frac{du}{dt} - C \frac{d^2u}{dx^2} = 0$$

$$\frac{du}{dt} = C \frac{d^2u}{dx^2}$$

$$U_{i,j+1} - U_{i,j} = C \frac{\Delta t}{\Delta x^2} [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} - U_{i,j} = \frac{C \Delta t}{\Delta x^2} [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U[x_0, 0] = x^4 k$$

$$\Delta x = 0.2m, \Delta t = 0.02 \text{ s}$$

For initial conditions

$$\Delta t \quad x=0 \rightarrow 0^4 = 0$$

$$\Delta t \quad x=0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\Delta t \quad x=0.4 \rightarrow 0.4^4 = 0.0256$$

$$\Delta t \quad x=0.6 \rightarrow 0.6^4 = 0.1296$$

$$\Delta t \quad x=0.8 \rightarrow 0.8^4 = 0.4096$$

$$\Delta t \quad x=1 \rightarrow 1^4 = 1$$

$$\therefore U_{i,j+1} = U_{i,j} + r [U_{i+1,j} - 2U_{i,j} + U_{i-1,j}]$$

$$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j}$$

when $i=1, j=0$

$$U_{1,1} = 0.5 U_{0,0} + 0.5 U_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$U_{1,1} = 0.0128$$

when $i=2, j=0$

$$U_{2,1} = 0.5(U_{1,0}) + 0.5(U_{3,0}) \\ = 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$U_{2,1} = 0.0656$$

when $i=3, j=0 = U_{3,1} = 0.5(U_{2,0}) + 0.5(U_{4,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{3,1} = 0.2176$$

when $i=4, j=0 = U_{4,1} = 0.5(U_{3,0}) + 0.5(U_{5,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$U_{4,1} = 0.5648$$

For $j=1$

when $i=1 \therefore U_{1,2} = 0.5(U_{0,1}) + 0.5(U_{2,1})$

$$= 0.5(U_{0,1}) + 0.5(U_{2,1}) = 0 + 0.5(0.0656)$$

$$U_{1,2} = 0.0328$$

when $i=2 \therefore U_{2,2} = 0.5(U_{1,1}) + 0.5(U_{3,1})$

$$= 0.5(0.0128) + 0.5(0.2176)$$

$$U_{2,2} = 0.1152$$

when $i=3 \quad U_{3,2} = 0.5(U_{2,1}) + 0.5(U_{4,1})$

$$= 0.5(0.0656) + 0.5(0.5648)$$

$$U_{3,2} = 0.3152$$

when $i=4 \quad U_{4,2} = 0.5(U_{3,1}) + 0.5(U_{5,1})$

$$= 0.5(0.2176) + 0.5(1)$$

$$U_{4,2} = 0.6088$$

For $j=2$

when $i=1 \therefore U_{1,3} = 0.5(U_{0,2}) + 0.5(U_{2,2})$

$$= 0 + 0.5(0.1152) = 0.0576$$

when $i=2 \quad U_{2,3} = 0.5(U_{1,2}) + 0.5(U_{3,2})$

$$= 0.5(0.0328) + 0.5(0.3152)$$

$$= 0.174$$

when $i=3 \quad U_{3,3} = 0.5(U_{2,2}) + 0.5(U_{4,2})$

$$= 0.5(0.1152) + 0.5(0.6088) = 0.362$$

$$\begin{aligned} \text{when } j=4 \quad U_{4,3} &= 0.5(3.2) + 0.5(U_{5,2}) \\ &= 0.5(0.3152) + 0.5(1) \\ U_{4,3} &= 0.6576. \end{aligned}$$

For $j=3$

$$\begin{aligned} \text{when } i=1 \quad U_{1,4} &= 0.5(U_{0,3}) + 0.5(U_{2,3}) \\ &= 0.5(0) + 0.5(0.174) = 0.084 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 \quad U_{2,4} &= 0.5(U_{1,3}) + 0.5(U_{3,3}) \\ &= 0.5(0.0576) + 0.5(0.362) = 0.2098 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 \quad U_{3,4} &= 0.5(U_{2,3}) + 0.5(U_{4,3}) \\ &= 0.5(0.174) + 0.5(0.6576) = 0.4158 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 \quad U_{4,4} &= 0.5(U_{3,3}) + 0.5(U_{5,3}) \\ &= 0.5(0.362) + 0.5(1) \\ &= 0.681 \end{aligned}$$

For $j=4$

$$\begin{aligned} \text{when } i=1 \quad U_{1,5} &= 0.5(U_{0,4}) + 0.5(U_{2,4}) \\ &= 0.5(0.2098) = 0.1049 \end{aligned}$$

$$\begin{aligned} \text{when } i=2 \quad U_{2,5} &= 0.5(U_{1,4}) + 0.5(U_{3,4}) \\ &= 0.5(0.087) + 0.5(0.4158) = 0.2514 \end{aligned}$$

$$\begin{aligned} \text{when } i=3 \quad U_{3,5} &= 0.5(U_{2,4}) + 0.5(U_{4,4}) \\ &= 0.5(0.2098) + 0.5(0.681) \\ &= 0.4454 \end{aligned}$$

$$\begin{aligned} \text{when } i=4 \quad U_{4,5} &= 0.5(U_{3,4}) + 0.5(U_{5,4}) \\ &= 0.5(0.4158) + 0.5(1) \\ &= 0.7079 \end{aligned}$$

| | | | | | | |
|------|---|--------|--------|--------|--------|---|
| 0.1 | 0 | 0.1049 | 0.2514 | 0.4454 | 0.7079 | 1 |
| 0.08 | 0 | 0.087 | 0.2098 | 0.4158 | 0.681 | 1 |
| 0.06 | 0 | 0.0576 | 0.174 | 0.362 | 0.6576 | 1 |
| 0.04 | 0 | 0.0328 | 0.1152 | 0.3152 | 0.6088 | 1 |
| 0.02 | 0 | 0.0128 | 0.0656 | 0.2176 | 0.5648 | 1 |
| 0 | 0 | 0.0016 | 0.0256 | 0.1296 | 0.4096 | 1 |
| | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |

0.5

