

Dr. D. W. B. S. Solomon  
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$$\frac{d^2 u}{dt^2} - c \frac{d^2 u}{dx^2} = 0$$

$$\frac{d^2 u}{dt^2} = c \frac{d^2 u}{dx^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$u_{i,j+1} - u_{i,j} = c \frac{\Delta t}{\Delta x^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}] \cdot \tau = \frac{\Delta t}{\Delta x^2}$$

$$u(x, 0) = x^4 k$$

$$\Delta x = 0.2 \text{ m}, \Delta t = 0.02 \text{ day}$$

for initial conditions

$$\text{At } x = 0 \rightarrow x^4 = 0$$

$$\text{At } x = 0.2 \rightarrow 0.2^4 = 1.6 \times 10^{-3}$$

$$\text{At } x = 0.4 \rightarrow 0.4^4 = 0.256$$

$$\text{At } x = 0.6 \rightarrow 0.6^4 = 0.1296$$

$$\text{At } x = 0.8 \rightarrow 0.8^4 = 0.4096$$

$$\text{At } x = 1 \rightarrow 1^4 = 1$$

$$u_{i,j+1} = u_{i,j} + \tau [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$$

$$u_{i,j+1} = \tau u_{i-1,j} + (1 - 2\tau) u_{i,j} + \tau u_{i+1,j}$$

(2)

When  $i=1, j=0$

$$u_{1,1} = 0.5u_{0,0} + 0.5u_{2,0}$$

$$= 0.5(0) + 0.5(0.0256)$$

$$u_{1,1} = 0.0128$$

When  $i=2, j=0$

$$u_{2,1} = 0.5(u_{1,0}) + 0.5(u_{3,0})$$

$$= 0.5(1.6 \times 10^{-3}) + 0.5(0.1296)$$

$$u_{2,1} = 0.0656$$

When  $i=3, j=0 \Rightarrow u_{3,1} = 0.5(u_{2,0}) + 0.5(u_{4,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$u_{3,1} = 0.2176$$

When  $i=4, j=0 \Rightarrow u_{4,1} = 0.5(u_{3,0}) + 0.5(u_{5,0})$

$$= 0.5(0.0256) + 0.5(0.4096)$$

$$u_{4,1} = 0.5648$$

for  $j=1$

when  $i=1 \Rightarrow u_{1,2} = 0.5(u_{0,1}) + 0.5(u_{2,1})$

$$= 0.5u_{0,1} + 0.5u_{2,1} = 0 + 0.5(0.0656)$$

$$u_{1,2} = 0.0328$$

when  $i=2 \Rightarrow u_{2,2} = 0.5(u_{1,1}) + 0.5(u_{3,1})$

$$= 0.5(0.0128) + 0.5(0.2176)$$

$$u_{2,2} = 0.1152$$

when  $i=3$   $u_{3,2} = 0.5(u_{0,1}) + 0.5(u_{4,1})$  (3)  
 $0.5(0.0656) + 0.5(0.5648)$   
 $u_{3,2} = 0.3152$

when  $i=4$   $u_{4,2} = 0.5(u_{2,1}) + 0.5(u_{5,1})$   
 $= 0.5(0.2176) + 0.5(1)$   
 $u_{4,2} = 0.6088$

for  $J=2$

when  $i=1$   $u_{1,3} = 0.5(u_{0,2}) + 0.5(u_{2,2})$   
 $= 0 + 0.5(0.1152) = 0.0576$

when  $i=2$   $u_{2,3} = 0.5(u_{1,2}) + 0.5(u_{3,2})$   
 $= 0.5(0.0328) + 0.5(0.3152) = 0.174$

when  $i=3$   $u_{3,3} = 0.5(u_{2,2}) + 0.5(u_{4,2})$   
 $= 0.5(0.1152) + 0.5(0.6088) = 0.362$

when  $i=4$   $u_{4,3} = 0.5(u_{3,2}) + 0.5(u_{5,2})$   
 $= 0.5(0.3152) + 0.5(1)$   
 $u_{4,3} = 0.6576$

when  $J=3$

when  $i=1$   $u_{1,4} = 0.5(u_{0,3}) + 0.5(u_{2,3})$   
 $= 0.5(0) + 0.5(0.174) = 0.087$

when  $i=2$   $u_{2,4} = 0.5(u_{1,3}) + 0.5(u_{3,3})$   
 $= 0.5(0.0576) + 0.5(0.362) = 0.209$

when  $i=3$   $u_{3,4} = 0.5(u_{2,3}) + 0.5(u_{4,3})$   
 $= 0.5(0.174) + 0.5(0.6576) = 0.4158$

When  $i=4$   $U_{4,4} = 0.5(U_{5,3}) + 0.5(U_{5,3})$   
 $= 0.5(0.362) + 0.5(1)$   
 $= 0.681$

When  $J=4$

When  $i=1$   $U_{1,5} = 0.5(U_{0,4}) + 0.5(U_{2,4})$   
 $= 0.5(0.2098) = 0.1049$

When  $i=2$   $U_{2,5} = 0.5(U_{1,4}) + 0.5(U_{3,4})$   
 $= 0.5(0.087) + 0.5(0.4158) = 0.2514$

When  $i=3$   $U_{3,5} = 0.5(U_{2,4}) + 0.5(U_{4,4})$   
 ~~$= 0.5(0.2098) + 0.5(0.4158) = 0.3128$~~

When  $i=4$   $0.5(0.2098) + 0.5(0.681) = 0.4454$

When  $i=4$   $U_{4,5} = 0.5(U_{3,4}) + 0.5(U_{5,4})$   
 $= 0.5(0.4158) + 0.5(1)$   
 $= 0.7079$

$\Delta t$	$J/T_{emp}(K)$					
0.1	5	0	0.1049	0.2514	0.4454	0.7079
0.08	4	0	0.087	0.2098	0.4158	0.681
0.06	3	0	0.0576	0.174	0.362	0.6576
0.04	2	0	0.0328	0.1152	0.3152	0.6008
0.02	1	0	0.0128	0.0656	0.2176	0.5648
0	0	0	0.0016	0.028	0.1296	0.4096
$\Delta x$		0	0.2	0.4	0.6	0.8
		0	1	2	3	4

0.1	0	0.1040	0.2514	0.4454	0.7079	1
0.08	0	0.087	0.2098	0.4158	0.681	1
0.06	0	0.070	0.174	0.362	0.6576	1
0.04	0	0.0528	0.1452	0.3152	0.6038	1
0.02	0	0.0356	0.1156	0.2716	0.5648	1
0	0	0.0184	0.0856	0.2284	0.4976	1
	0	0.2	0.4	0.6	0.8	1

0.5

